TRANSACTIONS OF
THE
OPTICAL SOCIETY

VOL. XXXI. 1929-30 No. 1.

THE INSENSITIVITY AND PERSONAL EQUATION ERRORS OF OPTICAL SETTINGS

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MS. received, 12th November, 1929. Read and discussed, 12th December, 1929.

ABSTRACT. The paper contains the results of observations on the insensitivity, that is, mean difference of the individual observations of a large series from the mean of the series, in the case of *X* type graticules set on vertical object lines.

The influence of the following factors was investigated:

(1) Thickness of cross-wires.
(2) Angle between cross-wires.
(3) Field brightness.
(4) Pupillary aperture.

Observations were also made of the magnitude of the "personal equation" error in such settings.

THE purpose of this work was to determine the influence of various conditions on the insensitivity and personal equation errors of settings such as are made in various optical instruments where a direction or position has to be determined by setting "cross-lines" in relation to the image of an object formed in the focal plane of the instrument.

One such setting, namely the coincidence setting of two sections of a broken line as employed in the coincidence type of rangefinder, has been investigated in considerable detail by J. W. French*. The types of setting more commonly used in optical instruments have not been investigated so systematically, and no quantitative information regarding personal equation errors appears to have been obtained.

The present paper deals with the type of setting chiefly employed in instruments of the spectroscope or goniometer type and in many surveying instruments, in

* *Trans. Opt. Soc.* 21 1919-20 (127).

which the object sighted is a thin vertical line, such as a spectrum line, the image of a staff, or other similar linear object, while the graticule consists of two diagonally mounted cross-lines, forming the letter X, the intersection of which is set on the vertical object line. It was intended that other types of setting, such as that in which the graticule consists of two close parallel vertical lines which are set so as to enclose, symmetrically, the object line, should be dealt with at the same time, but this was impossible owing to pressure of other work.

Method of Experiment

The insensitivity of setting with an optical instrument depends not only on the insensitivity of the eye, but also on the magnifying power of the optical system. It was decided to make all determinations with the unaided eye. The angular constants so obtained have only to be divided by the magnifying power of any telescopic system to give the values which should apply when using such a system in addition to the eye.

The simplest procedure for determining the insensitivity of the unaided eye is to construct the graticule and object line on a suitably large scale and arrange so that one is movable in front of the other at some distance from the observer. In practice, the drawback to such a simple arrangement lies in the fact that at any observation distance which is feasible in a laboratory experiment, very minute relative movements of the graticule and object are significant. For example, at a distance of 5 metres a relative movement of 0·024 mm. corresponds to an angle of 1 second. Very accurate construction of the carriage carrying the moving graticule, together with provision for accurately reading its position, would be necessary in order to preclude the possibility of instrumental errors of 1 second, an amount which is by no means negligible as the results show. Further, the provision of a mechanism for operating the moving part, at a distance of 5 metres, with the necessary delicacy, presents considerable difficulty in view of the weight of the framework required to carry the graticule. These mechanical difficulties were finally overcome by the following device: Instead of moving the graticule in front of a fixed object line, both graticule and object line were attached to the carriage so that they moved together. They were separated *in the direction of the line of sight* by a distance equal to one-hundredth of their distance from the observer, who viewed them from a definitely located eyepoint. There was therefore an apparent relative movement of cross-lines and object, due to parallax, equal to a hundredth of the actual movement of the carriage. This magnification of the actual movement required to produce a given angular alteration in the relative positions of graticule and object enabled the required delicacy to be obtained with relatively crude moving parts and simple activating mechanism, since an angular change of 1 second now required a movement of the carriage of 2·4 mm.

The actual arrangement employed at the graticule end of the apparatus is shown in the photograph, Fig. 1.

A flanged brass ring of 70 cm. diameter and 5 cm. deep was supported on a carriage, fitted with rollers, which moved on a short length of photometer bench. The cross-wires were attached to the front of the ring by thumb screws which could be inserted in any of a series of perforations in the flange, thus enabling the wires to be fixed at any desired angle.

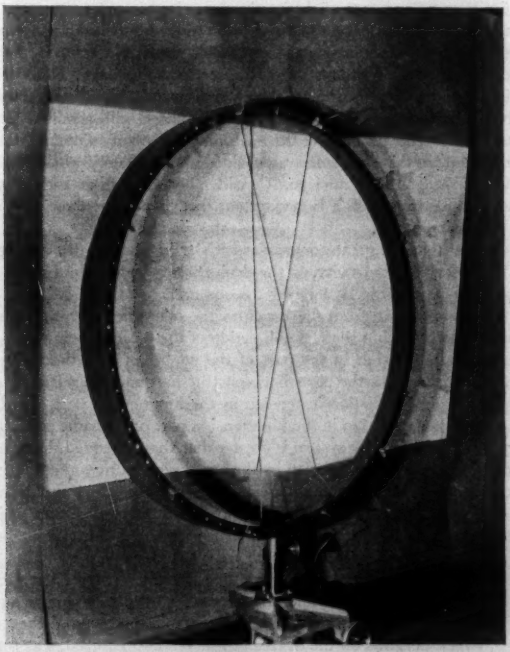


Fig. 1

The object wire was mounted vertically at the back edge of the ring, and was thus behind the plane of the graticule by the width of the ring, i.e. 5 cm. The object wire was kept taut by means of a short length of spiral spring to compensate for any slight deformation of the frame caused when mounting the graticule wires.

By suitable adaptation any type of graticule, other than the diagonal cross-wires so far tested, could be attached to the frame.

Field illumination was provided by a white paper diffusing screen illuminated from behind by a number of electric lamps so distributed as to give sensibly uniform brightness.

The Activating Mechanism

As quite large movements of the graticule carriage were necessary to produce sensible changes in the apparent state of the setting, no delicacy was required in the means provided for moving the carriage. This was first effected by pull, in one or other direction, of a continuous cord attached to the carriage. This cord passed round suitable pulleys attached to the bench and also round a drum convenient to the observer's right hand. This drum, which was fitted with a milled head 3 cm. in diameter, fulfilled the function of the micrometer drum of an optical instrument. The arrangement, in this simple form, provided an adequately sensitive control of the position of the graticule carriage. Considerably more force had, however, to be expended in moving such a heavy system than is usually associated with the operation of a micrometer.

It was feared that the greater muscular activity involved in the settings might influence the observer's sensitivity, so the device was adopted of inserting a mechanical relay to reduce the work done by the observer.

Both the cords leading from the bench to the micrometer drum were given a few turns round a revolving shaft, the direction of the rotation being such that the torque increases the tension in the part of the cords between the shaft and the bench and diminishes it in the parts between the shaft and the micrometer. With such a system a slight variation in tension in the low tension part of either cord produces a magnified variation in the high tension part of the same cord. Thus the energy supplied to the moving system is greater than that expended by the operator, the balance of the work being supplied by the motor which drives the rotating shaft. The magnification which may be obtained in this way increases rapidly with the number of turns of the cords round the shaft and is theoretically unlimited. In practice too much magnification cannot be used without the system becoming unstable, slight spontaneous changes which are unavoidable in the friction between the cords and shaft being sufficient to move the carriage. With two turns, however, the force required to operate the micrometer was reduced to a similar amount to that required to operate the average type of screw micrometer, and the "feel" of the settings, which depends on the degree of innervation required, was practically identical in the two cases.

In this way the possibility of reflex actions affecting the insensitivity of setting and rendering the results obtained not strictly comparable with those to be expected in ordinary optical work was eliminated.

The observer looked through a fixed eye-hole, 5 mm. in diameter. Movements of the head were prevented by a rubber mask, moulded to fit the forehead and nose, and adjusted by each observer so as to give central vision through the fixed eye-hole.

Mode of Investigation

While the results of investigations on insensitivity or personal equation errors must be statistical, in order to indicate with reasonable certainty the properties of the average observer, to apply statistical methods involving a number of observers to all the observations would necessitate a prohibitive expenditure of time. The plan was therefore adopted of investigating the qualitative aspects of the problem by one or two observers who, from general experience within the Optics Department of the Laboratory, were known to be fairly average observers, subsequently employing check observations by other observers to establish a more representative figure for the more important results.

As already indicated, the present paper deals with cross-lines of X form, set on a vertical line bisecting the top and bottom angles of the X.

The influence on insensitivity and personal equation errors of the following factors were investigated:

(1) Thickness of cross-wires.
(2) Angle between cross-wires.
(3) Field brightness.
(4) Pupillary aperture.

By "insensitivity" is here meant the average degree of departure of a single observation from the ultimate value which the observer would obtain with a sufficiently large number of observations to render accidental errors of observation negligible. By "personal equation error" is meant the difference between the observer's ultimate value and the true value of the setting, as defined by the geometrical relationship of the object and graticule to the direction of observation.

Preliminary experiments showed that the mean of a series of 25 settings gave an adequately close approximation to the observer's ultimate value for the purposes of this work. The average difference of the individual settings comprised in such a series from the mean of the series was taken as a criterion of the insensitivity of the settings under the conditions prevailing. All settings in a single series were made in the same direction, i.e. graticule approaching object from the right or left as the case might be. In alternate series the direction of approach was reversed.

To estimate the personal equation error the device was adopted of placing a telescope behind the observation pupil so as to obtain a magnified picture of the field of view. If m is the linear magnification of the telescope, all errors, whether observational or due to personal equation, are reduced to $1/m$ of their value with the naked eye, assuming that such errors are not affected by the increase in apparent size of the wires when viewed through the telescope.

As a matter of fact, with the magnification employed, about 25, the wires were seen in such clear detail that the correct geometrical setting could be located without the possibility of sensible ambiguity, and two or three settings sufficed to locate the "true reading" with sufficient accuracy.

The telescope was arranged so that it could be conveniently swung into or out of the line of sight without disturbing any other apparatus.

A true reading was taken before and after each series of 25 naked eye settings, and a series was rejected if for any cause the true reading had changed during the observations.

Effect on Insensitivity of Thickness of Cross-wires

Observations were made on a series of cross-lines ranging in angular width, as seen by the unaided eye, from 10 to 180 seconds.

In all cases the vertical object line was of the same width as the graticule lines.

These observations preceded the experiments on the best angle between the cross-lines, and an angle of 60° was employed, as being one fairly frequently used in optical instruments.

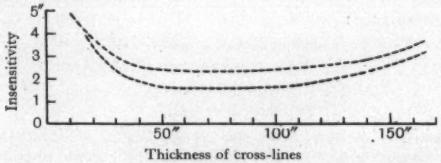


Fig. 2

In Fig. 2 the results for two observers are shown.

While the insensitivity differs in the two cases, there is marked similarity in the mode of variation with wire thickness. In each case the insensitivity varies very little with thickness over a wide range from about 40 to 120 seconds. With wires less than 40 seconds it increases rapidly. With wires more than 120 seconds it also increases, but much less rapidly.

It would appear, from these results, that the angular size of cross-wires in an optical instrument, i.e. the angle which the width of the wires subtends at the nodal point of the eyepiece, should be between 40 and 120 seconds to ensure the best conditions of sensitivity for settings of this type.

Effect of Insensitivity of Angle Between Cross-lines

By angle between cross-lines is here meant the angle of the two sectors of the field (usually the top and bottom sectors) within which the object wire lies when a setting is made. There is wide diversity of practice in the design of optical instruments as regards the mounting of cross-lines, and angles varying from 15° to 90° are found, a favourite value being 60°.

Fig. 3 shows the effect on insensitivity of angle of cross-lines for three observers. The wire thickness for all these experiments was 60 seconds. This is within the optimum range of Fig. 2, the particular value being chosen from considerations of

convenience as regards available material, a flat woven copper braid being used in preference to wire of round section.

It will be noted that all three observers show a preference for an angle in the neighbourhood of 40° to 50°, although the loss of sensitivity for angles within a fair range on either side of this is not great. Although cross-lines with a contained angle greater than 90° are never found in optical instruments, the measurements were carried up to 130° as a matter of interest.

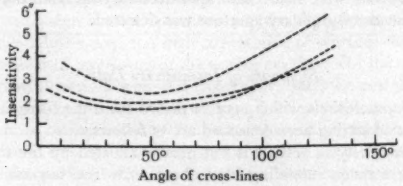


Fig. 3

Influence of Field Brightness on Insensitivity

The foregoing experiments were all carried out with a field brightness of approximately 5 candles per sq. ft. Experiments were made by a single observer to determine the influence of reduced luminosity on the insensitivity. Preliminary experiments indicated that the effect was negligible until quite low luminosities were reached. Careful comparisons were then made of the insensitivity for 5 and 0·05 candles per sq. ft. In order to ensure that the results were entirely comparable and not vitiated by the variations of an observer's sensitivity from one time to another, such variations being frequently as much as 100 per cent., the measurements at the two brightnesses were made concurrently, a high and a low brightness being employed for alternate settings, the reduction in brightness being affected by interposing a neutral filter, transmitting 1 per cent., in front of the eye. The two sets of readings so obtained were thus strictly comparable in all respects excepting field brightness.

As a result of such measurements the insensitivity at 0·05 candle per sq. ft. was found to be slightly greater than that at 5 candles per sq. ft., the mean difference from the means, in the two cases, being in the ratio 1·20 : 1.

It will be noted that the above procedure does not permit of the eye becoming fully adapted to the lower illumination, owing to the alternate exposures to the brighter field. It is very probable that the eye, if completely adapted to the lower illumination would, owing to the higher apparent brightness which the field would then have, give better sensitivity than under the conditions of the experiment.

It was not feasible, however, to determine the small differences of sensitivity involved by separate tests under conditions of complete adaptation to each field brightness, for the reasons stated, without a prohibitive amount of repetition of

observations. The figure given may therefore be taken as an *outside limit* for the loss of sensitivity at the lower illumination.

Effect of Pupillary Aperture on Insensitivity

All the foregoing observations were made with an observation pupil of 5 mm. As the pupillary aperture in many optical instruments is much less than this, series of observations were made with apertures varying from 0·5 mm. to 5 mm. No certain influence of pupillary aperture was detected.

Summary of Insensitivity Data

The general conclusions which may be drawn from the foregoing results concerning the type of setting here discussed are as follows:

1. The insensitivity of setting is not greatly affected by the thickness of the cross-wires over a range extending from about 40 to 120 seconds (angular width as seen by the eye) but increases seriously for wires thinner than 40 seconds and less seriously for wires thicker than 120 seconds. A desirable thickness to aim at where possible would be between 60 and 90 seconds.

2. The insensitivity is smallest when the angle between the cross-lines, as defined earlier, is about 45°, but the increase is slight for any angle not too far removed from this, say between 25° and 70°.

3. The insensitivity is practically independent of field brightness for all ordinary illuminations, and increases by not more than 20 per cent. for low illuminations of the order of 0·05 candle per sq. ft.

4. The insensitivity is independent of pupillary aperture from 0·5 mm. upwards, at any rate when optimum conditions prevail as regards the other factors, wire thickness, field brightness, etc.

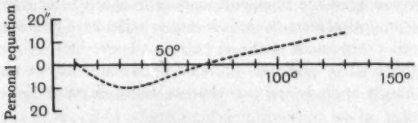
5. As an indication of the insensitivity of this type of setting under optimum conditions, the average difference of a single observation from the mean of a large number of observations is of the order 2 to 2½ seconds. To obtain this figure the observations of the observers responsible for the main qualitative parts of the work were supplemented by observations made from time to time by several additional observers under the optimum conditions deduced above from curves 2 and 3. The figure is the mean obtained under such conditions by seven observers of varying degrees of experience, in some cases very little, of such observations. The insensitivity for any one observer, whether experienced or not, varies from one occasion to another, by as much as 100 per cent., variations of 30 per cent. being frequent.

When employing a telescope of linear magnification m, the insensitivity of setting on a linear object, the image of which is comparable in sharpness and width with the cross-lines, should, from the foregoing results, be such that the average difference of a single observation from the mean of a large number is, say, $2{\cdot}5/m$ seconds, for an average observer.

Personal Equation

All the observations made for the insensitivity determinations were accompanied by measurement of the personal equation error, as described earlier. In the case of one observer this error was negligible in all the observations made. Both the other observers who made systematic tests with different angles of cross-lines, etc., showed personal equation displacements which were usually in excess of the observational error. These displacements were generally, though not invariably, of the same sign in whichever direction the setting was made. That is to say, the settings would be, say, too high irrespective of whether the settings were made in the direction of increasing or decreasing reading. On the other hand, the readings in the two directions were not usually identical, the personal equation in the two cases, though of the same sign, being appreciably different in magnitude.

The outstanding characteristic of the personal equation data obtained in these measurements was their variability from one occasion to another. It is quite unsafe to assume the same personal equation to hold for two similar sets of observations made at different times. A particular equation appears to be retained so long as observations are in more or less continuous progress, but an interregnum of any very great duration, such for instance as adjournment for lunch, or for the per-



Angle of cross-lines

Fig. 4

formance of other work, may suffice to destroy the "memory" of the setting and a different equation will generally be found on resumption of observation.

Thus in work with this type of graticule, the two sets of observations, the difference of which is the quantity to be measured, must be made concurrently if possible, or at least as nearly together as may be feasible, in order that the elimination of personal equation from the result can be relied on.

As regards the magnitude of the personal errors recorded, and their dependence on the variables investigated in this work, no systematic dependence on any of these variables emerges from the observations. A very rough dependence on the angle between the cross-wires appeared to hold for one observer. This is shown in Fig. 4. The figures obtained by the other observer, while possibly tending to exhibit a similar relationship, were too irregular to permit of drawing a curve.

The only generalisation which emerges from all the observations made, including those of observers whose observations, as already mentioned, were confined to the neighbourhood of optimum conditions as regards sensitivity, is that personal equation errors under such conditions do not *frequently* exceed about 5 seconds, although they may, on occasion, be as high as 10 or 12 seconds.

DISCUSSION

Mr J. Rheinberg: Mr Guild's admirable paper has interested me very much. It may interest Mr Guild to have some confirmatory evidence of his results, and I should therefore like to say that, without having carried out any research work on the subject such as he has described, the result of many practical experiments with graticules which I have had occasion to make have led me to the same conclusions, viz. that within a wide range the thickness of cross-lines makes no difference to the sensitivity of settings, but that one has to guard against having the lines too fine. Also that within wide limits the brightness of the field makes no difference, until we come to a very low order of brightness.

I should like to ask Mr Guild whether he has made any experiments with semi-transparent lines, as I am inclined to believe that greater sensitivity is obtainable in appropriate cases by the use of lines which are semi-transparent. Not, naturally, in all cases; in the spectroscope, for example, we are already dealing with coloured lines which afford contrast.

Also, I should be glad to know in what way the observer's eye was fixed in Mr Guild's experiments. Questions of parallax can be rather important, because parallactic effects are possible even when (by reason of accommodation of the eye) an object image and graticule lines both appear in sharp focus. In the case of a graticule *scale* in an optical instrument it is easy to make sure of absence of parallax by focussing until a movement of the eye across the eye-lens results in no visible movement of the relative positions of scale marks and object marks, but this method would hardly apply to the experiments under discussion.

Capt. T. Y. Baker: The interesting account given by Mr Guild supplies further evidence that the opinion once held that the eye cannot appreciate angles of less than one minute of arc is not correct.

In measurements made in other forms of setting, notably in the coincidence setting used in range-finders, it is certain that the value of the insensitivity obtained depends upon the skill of the observer, and that, whereas a figure of 10 seconds is about right for the average man, the really skilled observer can do very much better. I should be glad if Mr Guild would give some information concerning the relative skill of the various observers employed in these experiments.

I hope that further measurements on other types of setting will be made. The passage of a bright star across successive spider lines, the reading of a single line against a staff as used in tacheometric surveying, and the "bisection" setting in which one moderately broad dark band has to be set midway between two parallel wires, are all forms of setting in which an accurate knowledge of the insensitivity of the eye would be of great value to instrument designers.

Mr A. Whitwell: I should like to ask Mr Guild whether his observers were emmetropes, and if not whether they wore their correcting lenses when making the observations? It seems to me that some of the faulty settings might be explained by the presence of perhaps small amounts of astigmatism in the observing eyes.

Mr T. Smith: The results given by Mr Guild in his paper contain many points of interest, and we should all like to see them extended to other types of object. It would be of interest, for example, to have the corresponding figures for bright lines on a dark ground and to know more about the importance of contrast. Comfort may be an important factor in settings of high accuracy—it is at least a factor in an observer's satisfaction with his readings.

I should be glad to know whether Mr Guild has formed any hypothesis which would furnish an explanation of the magnitudes obtained and elucidate the factors on which accuracy depends. Is the observer conscious of the criterion on which he relies? I recollect that the late Mr S. D. Chalmers considered that in a symmetrical setting the criterion is essentially photometric in character. This could perhaps be put to the test by obtaining figures for settings in which asymmetry is introduced by causing the single line to make a decidedly smaller angle with one of the cross-lines than with the other. A complete theory of all these acute powers of the eye would be of value. The difference in magnitude between the precision of setting and the resolving power of the eye is well known and is significant, but I think that in a setting of this type the resolving power of the eye is a pertinent factor. The falling off in the accuracy of setting when the thickness of the lines falls appreciably below 1 minute may be an illustration of the influence of this factor. Our attention is presumably directed to the region where the light between the three wires is falling markedly in intensity. The recognition that the wires are present is of course independent of resolution and is a simple matter of contrast. When, however, we come to consider the limit at which spots of light bounded by dark lines subtending a small angle at the eye are recognised as distinct sources, I suggest we are in the field where resolving power is by no means a negligible factor. A thorough discussion of the whole group of phenomena, utilising observations on a wide variety of settings under various conditions of illumination (including self-luminous wires) should be of great value to several groups of workers interested in the properties of the human eye.

Major E. O. Henrici: Capt. Baker has already said much that I intended to say. I hope Mr Guild will be able to make more measurements of the same type, and I suggest some trials to see the best angle for the cross-lines when the object sighted is appreciably thicker than the cross-lines.

In regard to Mr Smith's remarks, I do not think the intersection of the cross-lines is observed in a case where a thick line is observed by cross-lines with a rather small angle. It is then a question of symmetry.

The President (Mr F. Twyman): Mr Guild's paper affords an instance of a subject which appears at first sight uninteresting, but proves on more intimate acquaintance very interesting indeed. The author has dealt with his problem so thoroughly that anyone who wishes to measure the position of a black line by the movement of a black cross-wire need never in the future be in any kind of hesitation or doubt as to the dimensions which will yield him the most accurate result.

The accuracy possible in making measurements of the kinds referred to under the most favourable circumstances appears from Mr Guild's paper to be 2½ seconds. Since the effective resolving power of the eye corresponds to something like 1 minute, the accuracy of making a setting of this character is about 24 times the resolving power of the eye.

It may be interesting to mention the mode of setting adopted by the late Dr C. V. Burton, but never so far as I am aware published, by which a considerably higher accuracy of setting appears to be attainable. In the case dealt with by Dr Burton it was desired to obtain the greatest possible accuracy of measurement of the deflection of a galvanometer mirror. This was achieved by Dr Burton by using the galvanometer mirror to produce a much enlarged image of a brightly illuminated slit, and to obscure the image of the line of light thus formed by a thick wire, so that only the margins of the diffraction disc projected on each side of the wire. The two small illuminated regions thus visible on each side of the wire were adjusted to equality, and in this way Dr Burton informed me a measurement could be made to within 1/100th of the resolving power.

Every optical instrument maker should be grateful to Mr Guild for bringing one more problem from the region of taste into that of measurement.

Mr J. Guild (partly communicated): In reply to Mr Rheinberg, I am glad to see that his experience confirms the conclusions of the paper regarding the relatively slight influence of a wide range of variation of thickness of cross-lines and field brightness. I have not made observations with semi-transparent lines, though I should be doubtful if any advantage would accrue from their use. It may be worth mentioning here, though not specifically related to the work described in the paper, that the most satisfactory conditions for work of this kind are obtained when the cross-lines *and* the object line are dark against a bright field. If the object line is bright against a dark background the conditions of seeing are much less comfortable, even if bright cross-wires are also used, and the accuracy falls off accordingly. This replies also to part of Mr Smith's remarks. I described some years ago, in a paper to the Physical Society dealing with chromatic parallax*, a method for obtaining the best conditions, namely, a black line against a bright background, in the case of spectra of all but the greatest complexity. This consists of employing a wide slit, say 0·5 mm. or more, with a fibre similar to that used in graticules, or a black line on glass, as made by Mr Rheinberg, mounted along the centre of the slit. The broad band obtained for each spectral line provides the background against which the black central line and the telescope cross-wires are clearly contrasted. This gives much more accurate settings in visual measurements on spectra, such as are involved in refractive index measurements, for example, than the ordinary procedure of setting cross-wires on the bright image of a fine slit, and is always used for such work at the National Physical Laboratory.

As regards prevention of parallax in the work of the present investigation, by an oversight I omitted to include in the draft of the paper a description of the

* *Proc. Phys. Soc.* **29** 1917.

arrangements at the observer's eye. I have inserted this in the text when revising the proof.

In reply to Capt. Baker, the other types of setting to which he suggests attention should be given were all included in the programme which it was hoped to carry out about the same time as the work described, but were unfortunately ruled out for the time being owing to the relative urgency of other investigations.

This also covers some of Major Henrici's remarks. I am indebted to Major Henrici for sending me a reference to the work of Rheinhertz, which has special application to the reading of levelling staves, but have not yet had access to the publication in question.

In reply to Mr Whitwell, none of the observers use glasses in their daily occupations. Whether they ought to have worn them I am unable to say.

The President's reference to Dr Burton's device for improving the accuracy of reading the position of a galvanometer spot is of interest. The gain obtainable by this method, in a case such as that in question where the optical definition obtainable is very poor as compared with that in any optical instrument, and the resolving power is extremely low, is probably very much greater than would be obtained by its use under the conditions applicable to optical measurements.

With respect to the theoretical questions raised by Mr Smith and Capt. Baker, I think it is certain that resolving power is not a direct factor in the precision of optical settings. By this I mean that the probable error of setting, expressed as an angular separation of the object and the significant feature of the graticule, is not related to resolving power. The extreme smallness of the probable error as compared with resolving power renders a relationship unlikely, and the further fact that with optimum conditions as to wire thickness the insensitivity is apparently independent of pupillary aperture, makes its independence of resolving power, which varied over a range of 5 to 1 in these experiments, certain.

The suggestion of Mr Smith, that the falling off in sensitivity when the wire thicknesses were reduced below about 50 seconds is related to the question of resolution, in no way affects the foregoing conclusion; because the effect of resolving power which here becomes significant is not related to the power to resolve the separation between object and graticule, but to the influence of resolving power on the visibility of the wires. We have to regard an opaque wire seen against a bright background as a space separating two bright areas. It is really the bright areas we see, not the wire. If the angular width of the latter is greater than about a minute (in round figures) the bright areas on each side of it are completely resolved, and a clearly visible black line is seen along the centre of the geometrical shadow of the wire. When, however, the wire is narrower than this the images of the field on either side of its boundary are not properly resolved. There is overlapping of the diffracted light, and the contrast between the image of the wire and the field is reduced with consequent loss of visibility and comfort of seeing and a corresponding loss of sensitivity. As the width of the wire is reduced still further this effect rapidly increases until, below a certain width, the wire is invisible altogether owing to the complete failure to resolve the bright areas on either side of it. It is this

progressively increasing difficulty of seeing the wires, and the increased degree of enervation involved in keeping them clearly in view, which accounts for the rapid loss of sensitivity for wires narrower than about 50 seconds in Fig. 2.

I have long been familiar, of course, with Mr Chalmers' theory that the criterion in these settings was photometric in character, but have never been able to reconcile it with my own experience with a variety of different types of setting. The present paper only deals of course with one type of setting for which a systematic study was made; but there are many other types of setting which have been used for one purpose or another at the N.P.L., and although none of them have been studied in the present manner one knows quite well from accumulated experience that all of them give a sensitivity of the same order if used under critical conditions. Wide differences in character, as regards the distributions of light and dark regions, exist in such different types of setting as, for instance, that in which one line is made to bisect the narrow space between two others; settings such as that dealt with in this paper; and settings, such as I have frequently used, and have described in a paper* dealing with a special method of refractometry, in which the graticule consists of a pair of diagonal cross-wires (of the type of the present paper) and the object consists of a similar set of cross-lines slightly displaced vertically with respect to the graticule wires, the aim of the observer being to obtain equality in the four sides of the small parallelogram formed by the wires. It is difficult to account for the very similar sensitivities obtained with these various settings on a photometric theory, the factors which would enter into any feasible photometric criterion being so dissimilar.

The one feature which is common to all types of setting possessing high precision of which I have any knowledge is the symmetry of the picture presented to the eye when a satisfactory setting has been made; and it seems to me that the real criterion in all such settings, irrespective of what the observer may *think* he is aiming at, is the satisfaction of the sense of symmetry.

To develop this theory here would require more space than can fittingly be devoted to it in the reply to discussion on a paper of which the primary interest was intended to be practical rather than theoretical, but I am convinced that it is the only one which fits the wide group of facts involved.

* *Trans. Opt. Soc.* **22** 1920 (143); also Glazebrook's *Dictionary of Applied Physics*, **4**, article on "Spectroscopes and Refractometers."

THE REFLECTION FACTOR OF MAGNESIUM OXIDE

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[Communicated by Dr J. W. T. Walsh]

MS. received, 16th November, 1929. Read and discussed, 16th January, 1930.

ABSTRACT. This paper describes the design and use of a small integrating reflectometer for determining the total reflection factor of a smoked magnesium oxide surface under diffuse illumination. It also shows how the reflectometer may be used for the absolute determination of apparent reflection factors (reflectances) under parallel illumination.

The instrument is based on theory given by A. H. Taylor, *J.O.S.A.* **4**, 1920 (14), and depends on the change in brightness of the walls of a small integrating sphere which occurs on removal of a test plate covering a circular aperture of known area in the sphere. The theory is extended to show how the interior of the sphere may be used as a standard of brightness when the diffuse reflection factor of the coating and the flux entering the sphere are known. This affords a new means for the standardisation of photometric test plates. A second method of determining the diffuse reflection factor of the reflectometer coating is described depending on measurements of the brightness of the interior of the sphere and of an artificial sky used to illuminate it through the aperture.

In practice it is shown that the surface on which the oxide is smoked may influence the results except in the case of very thick coatings. Polished silver was finally selected for this purpose and the results given are for coatings on this surface. They include the total reflection factor under diffuse illumination and the apparent factor for normal incidence, 45° angle of view, and 45° incidence, normal view.

Finally the Helmholtz reciprocal relations as developed by H. J. McNicholas, *B. of S. Journal of Research*, **1**, No. 1, are utilised to calculate the apparent factors at normal and 45° angle of view and diffuse illumination from polar measurements of brightness under conditions of parallel illumination.

Introduction

The work described in the following paper applies to smoked surfaces of magnesium oxide only, as distinct from blocks of the oxide. It developed from the attempt to produce a type of reflectometer suitable for the determination of reflection factors under diffused light, and which would not be open to serious objections on theoretical grounds. The type selected involved a certain amount of work on the behaviour of the coating of a small integrating sphere, in this case magnesium oxide, and it was decided to utilise the reflectometer to extend this work and determine

accurately the reflection factor of the oxide under various conditions, including if possible parallel as well as diffuse incidence.

While the various methods hitherto described have been almost unanimous in the use of a spherical illuminator, they have also generally involved direct observation of the test specimen. This is open to two objections. First, if the specimen is viewed at one angle only, we learn nothing regarding its diffusing properties and have not sufficient data to obtain the total reflection factor. Second, the labour involved in overcoming the first objection by obtaining the polar curve of brightness of the specimen can hardly be justified when, as is often the case, the total reflection factor is of primary interest. In any case, this polar curve can only conveniently be obtained when the specimen is of such a size and shape as can conveniently be mounted inside a spherical illuminator*.

A. H. Taylor† has described a method in which all brightness measurements are made on the walls of the illuminating sphere, and which depends on the relative change in brightness of the walls which occurs on removal of a test plate covering a hole in the sphere. Briefly the procedure is as follows. The total diffuse reflection factor‡ of the interior coating of the sphere is obtained by calculation from the ratio in which the brightness is decreased when a standard plate covered with a coating identical with that of the sphere is removed. Knowledge of this ratio, together with a similar measurement when a test specimen of unknown reflection factor takes the place of the standard plate, suffices to determine the factor for the test specimen. While the accuracy of the latter determination may in certain circumstances be low, the accuracy of the determination of the reflection factor of the sphere coating may be very high for a coating of high factor, say greater than 90 per cent. This is due to the large concentration of flux inside the sphere which occurs when the hole is covered by the standard plate, and the correspondingly large increase in measured brightness.

This method, then, was adopted for the determination of the total diffuse reflection factor of magnesium oxide, the oxide being smoked on to the interior of the sphere and on to the standard plate. The theory, with modifications to allow for errors introduced in practice, is given below, but before proceeding with it and thence to the final determination of the reflection factor, some remarks concerning the preliminary experience obtained with the apparatus may not be out of place.

Measurements of the reflection factor of certain opal glass specimens showed that while the same coating of oxide remained on the reflectometer the results of measurements could be repeated within small limits of error. After recoating the sphere and standard plate and recalibrating, slightly different values were obtained, sufficiently far, however, from the first set of readings to lie outside the estimated limits of error on those readings. This pointed to an unsuspected error in the value determined for the oxide and drew attention to the possibility of unevenness in the coating, or of the standard plate receiving a coating of different thickness and re-

* J. M. Waldram, *I.C.I. Proc.* 7, 1928 (1020).
† A. H. Taylor, *J.O.S.A.* 4, 1920 (14).
‡ I.e., the total reflection factor under diffuse illumination.

flection factor from that on the sphere. The practice had been to give the reflectometer and plate two coats of a hard matt white enamel and smoke the oxide on to this surface. On examination the coating thus obtained was found to be much less dense and coherent than that which could be obtained directly on the bare copper shell of the sphere. Accordingly the enamel was removed and the copper cleansed and polished. The coatings of oxide obtained on this polished surface showed an increased reflection factor in addition to being distinctly harder than those obtained on the enamel. The following results were obtained, each figure being the mean of at least two determinations:

MgO on matt white enamel	MgO on polished copper
·93$_5$	·95$_5$
·93$_5$	·95$_8$
·94$_0$ (·94$_0$ by alternative method)	·96$_0$
·94$_5$	·95$_0$ (·94$_5$ by alternative method)
·94$_0$	·95$_0$ (·94$_5$ by alternative method)
·94$_5$	
Mean ·940	Mean ·951

The alternative method referred to in the table is one which was used as a check on the original method. It consisted in illuminating the interior of the sphere, via the test aperture, by means of an artificial sky. Comparison of the sky brightness with the brightness of the interior of the sphere enabled the reflection factor to be calculated. The value of this method as a confirmatory one lay chiefly in the fact that the flux entering the sphere was distributed diffusely, whereas in the first method it consisted of a narrow almost parallel pencil falling normally upon the sphere coating. It may be noted that the agreement between the two methods was good, and later became even better when coatings of more uniform reflection factor were obtained. The full theory of the alternative method is given later.

Subsequent deliberate efforts to increase the thickness of the oxide coating by prolonged smoking resulted in raising the reflection factor to about 97 per cent., showing that there was still room for considerable error due to unevenness in the coating. It also appeared that the chief factor affecting the measured absorption of light was really the backing on which the oxide was coated. Unless the coating was thicker than a certain limiting value light penetrating the coating was partially absorbed in the copper base. It was then decided to silver-plate the copper and thus to produce a backing which would be durable and also have a high reflection factor, approaching that of the oxide itself. Subsequent measurements of the factor for the polished silver gave values in the neighbourhood of 92 per cent. Coatings of oxide on this silver surface gave a reflection factor of about 97·4 per cent. and, what is more important, were easily reproducible provided that a thickness of roughly 1/3 mm. at least was aimed at. At this stage the reproducibility of the surface was considered satisfactory and it was decided to proceed with the accurate determination of its reflection factor for the standard conditions of parallel as well as diffuse illumination.

Theory

The theory given by A. H. Taylor* takes no account of the apertures in the reflectometer sphere which are necessary for admission of light and for the fixture of the observing eyepiece. For accurate work a correction for the loss of flux through these apertures is desirable, both in this method and in the method employing an artificial sky. The following preliminary calculation is used in both cases.

Let us assume we have a sphere the interior of which has a uniform brightness b. The sphere has two apertures of plane areas A and B, spherical areas A_s and B_s. The flux escaping through these apertures is made up as follows:

Flux lost = flux escaping through A assuming rest of sphere complete

\+ „ „ B assuming rest of sphere complete

− „ „ A from spherical cap equivalent to B_s

− „ „ B from spherical cap equivalent to A_s

$$= \pi bA + \pi bB - \pi b\frac{A_sB_s}{4\pi R^2} - \pi b\frac{B_sA_s}{4\pi R^2},$$

where R is the radius of the sphere.

If A, B, A_s and B_s are areas relative to the total sphere area the equation becomes

$$\text{Flux lost} = \pi b\,[A + B - 2A_sB_s]\,4\pi R^2.$$

Similarly if there are three apertures A, B and C we have

$$\text{Flux lost} = \pi b\,[A + B + C - 2A_sB_s - 2B_sC_s - 2C_sA_s]\,4\pi R^2.$$

Supposing, as is the case in the reflectometer to be described, two apertures, B and C for instance, are small, we can omit the term $2B_sC_s$ and rewrite the expression thus:

$$\pi b\,[A + (B + C) - 2A_s\,(B_s + C_s)]\,4\pi R^2,$$

which has the same form as that for two apertures. We then can add the areas of the small apertures and proceed as though they were one.

Diffuse Reflection Factor

Method I. The reflectometer consisted of a copper sphere with three circular apertures whose centres lay on three mutually perpendicular axes. Two of these apertures, used for illuminating and viewing the interior of the sphere, were small and are treated as one aperture, while the third was larger and is treated separately.

Let c' = spherical area of test aperture.

„ c'' = plane area of test aperture.

„ c_1 = sum of areas of observing and illuminating apertures. [Plane area = spherical area approximately.]

* *Loc. cit.*

Let $a = 4\pi R^2 - c' - c_1$.

„ m = total diffuse reflection factor of sphere surface and standard plate.

„ F = flux received through illuminating aperture.

„ b_0 = mean brightness of sphere wall *for reflected light only*, test aperture uncovered.

„ b = ditto, test aperture covered with standard plate.

Note. The apertures represented by c_1 are assumed to have zero reflection factor.

1. *Test aperture uncovered.*

First reflected flux escaping from sphere $= mF(c' + c_1)/4\pi R^2$.

Multiple-reflected flux escaping $= \pi b_0 (c'' + c_1) - 2\pi b_0 c' c_1/4\pi R^2$.

Flux absorbed in sphere coating $= (1 - m)\left[\frac{\pi b_0 a}{m} + F\right]$.

Hence, equating the sum of these to the entering flux:

$$F = mF(c' + c_1)/4\pi R^2 + \pi b_0 (c'' + c_1) - 2b_0 c' c_1/4\pi R^2 + (1 - m)\left[\frac{\pi b_0 a}{m} + F\right],$$

and

$$b_0 = \frac{m^2 Fa/4\pi R^2}{\pi\left[m\left(c'' + c_1 - \frac{2c' c_1}{4\pi R^2}\right) + a(1 - m)\right]},$$

or, if we consider the areas as being relative to the total sphere area instead of absolute,

$$4\pi R^2 . b_0 = \frac{m^2 Fa}{\pi[m(c'' + c_1 - 2c' c_1) + a(1 - m)]} \qquad \text{.....(1).}$$

2. *Test aperture covered with standard plate.*

Flux received by plate $= mFc'/4\pi R^2 + \pi b\left[c'' - \frac{c' c_1}{4\pi R^2}\right]$.

Flux received by sphere walls $= \frac{\pi ba}{m} + F$.

∴ Flux absorbed in the coating

$$= (1 - m)\left[\frac{mFc'}{4\pi R^2} + \pi b\left(c'' - \frac{c' c_1}{4\pi R^2}\right) + \frac{\pi ba}{m} + F\right].$$

Flux escaping through c_1

$$= mFc_1 + \pi b\left(c_1 - \frac{c' c_1}{4\pi R^2}\right) \text{ from sphere walls}$$

$$+ m\left[\frac{mFc'}{4\pi R^2} + \pi b\left(c'' - \frac{c' c_1}{4\pi R^2}\right)\right]\frac{c_1}{4\pi R^2} \text{ from test surface.}$$

Hence, as before,

$$F = (1 - m)\left[\frac{mFc'}{4\pi R^2} + \pi b\left(c'' - \frac{c' c_1}{4\pi R^2}\right) + \frac{\pi ba}{m} + F\right] + m\left[\frac{mFc'}{4\pi R^2} + \pi bc''\right]\frac{c_1}{4\pi R^2}$$
$$+ \pi b\left(c_1 - \frac{c' c_1}{4\pi R^2}\right) + mFc_1,$$

omitting a term in c_1^2.

Whence

$$b = \frac{m^2 F\,[a + c'm\,(1 - c_1)]/4\pi R^2}{\pi\left[(1-m)\left(a + c''m - \frac{c'c_1 m}{4\pi R^2}\right) + \frac{c_1 m}{4\pi R^2}(mc'' + 4\pi R^2 - c')\right]}$$

or, using relative areas,

$$4\pi R^2 . b = \frac{m^2 F\,[a + c'm\,(1 - c_1)]}{\pi\,[(1-m)\,(a + c''m - c'c_1 m) + c_1 m\,(mc'' + 1 - c')]} \quad \ldots(2).$$

The ratio $b : b_0$ is measured and the dimensional constants are determined from the apparatus, hence m can be obtained by utilising the equations (1) and (2).

Note. In the above theory the same factor m is used for the first reflection of the entrant pencil of light as for the subsequent diffuse reflections. This is inexact, but the agreement of the results with those given by Method II is a justification for this simplification, which is unlikely in any case to involve a large error.

Method II. The test aperture was covered with a piece of dense ground pot opal forming the base of a diffuse illuminator. This was used as an artificial sky. In the steady state the flux entering the sphere from the sky must be equivalent to that returned to the sky from the sphere, plus the loss by absorption in the sphere coating. The following theory then holds:

Let b_0 = sky brightness,
„ b = sphere brightness,
„ m = total diffuse reflection factor of sphere walls.

Then Flux entering sphere $= \pi b_0 c''$.

$$\text{Flux escaping} = \pi b\left(c'' + c_1 - \frac{2c'c_1}{4\pi R^2}\right) + \pi b_0 \frac{c'c_1}{4\pi R^2}.$$

$$\text{Flux absorbed} = \frac{\pi ba}{m}(1 - m).$$

Hence $$b_0 c'' = b\,(c'' + c_1 - 2c'c_1) + b_0 c'c_1 + ba\left(\frac{1}{m} - 1\right),$$

replacing absolute by relative areas for simplicity. Knowledge of the dimensions together with measurement of b_0/b enables us to calculate m.

Direct Reflection Factor

Method I. The sphere, as used for the measurement of diffuse reflection factor, was set up with the test aperture in the same position as would normally be occupied by a test plate and was illuminated by a concentrated filament lamp at a suitable distance to render the divergence of the incident light as small as was conveniently possible. A measure of the brightness of the interior of the sphere (at a part not directly illuminated) was then made. The aperture in the sphere was then replaced by the standard plate used in the previous work and a measure of its brightness

made at a suitable angle of view, the lamp remaining fixed. In the first case, by (1) above we have

$$\text{Brightness } b_0 = \frac{m^2 Fa}{\pi\,[m\,(c'' + c_1 - 2c'c_1) + a\,(1-m)]\,4\pi R^2}$$

$$= \frac{m^2 f c'' a}{\pi\,[m\,(c'' + c_1 - 2c'c_1) + a\,(1-m)]} \qquad \ldots\ldots(3),$$

if f = flux per unit area entering test aperture.

In the second case, if ρ is the apparent reflection factor* of the standard plate in the selected direction of view, we have

$$\text{Brightness } b' = \rho f/\pi \qquad \ldots\ldots(4).$$

As we already know m, the ratio $b' : b_0$ eliminates "f" and gives ρ.

Note. For tests under oblique incidence both plate and sphere were set up accurately at the same angle of incidence to avoid introduction of a cosine term. The source of light was at a distance sufficient to avoid too great an unevenness of illumination.

Method II. As a check on the first method the oxide surface was compared directly with a sheet of matt white opal glass standardised in the usual way by a comparison of the candle-power of a known area with that of the lamp used to illuminate it.

Apparent Reflection Factor in Diffused Light

The apparent reflection factor of magnesium oxide at different angles of view when illuminated diffusely is a matter of interest in colorimetric practice. The angles of view most widely in use are two, viz., normal and 45°.

H. J. McNicholas† has shown how the reciprocal relations governing the incident and reflected flux, first formulated by Helmholtz, may be employed to deduce a connexion between apparent reflection factors under diffused light and total factors under parallel incidence. In particular they enable us to calculate the apparent reflection factor under the conditions just described provided we know completely the behaviour of the surface under parallel illumination directed respectively along the normal or at 45° to it. Briefly, the total reflection factor for a given angle of incidence is equal to the apparent factor for the same angle of view but with diffuse incidence. If then we have available polar curves of relative brightness which represent the distribution of reflected flux under parallel illumination we can obtain a value for the integral reflected flux in relative measure. If in addition we know the apparent reflection factor for a definite angle of view, we have all the data necessary for the calculation of the total reflection factor. For simplicity let us consider the case of normal incidence, giving a polar surface of brightness which is symmetrical about the normal.

* Defined as the ratio of the brightness of the specimen to that of a perfect diffuser of zero absorption under the same illumination.

† *B. of S. Journal of Research*, 1, No. 1.

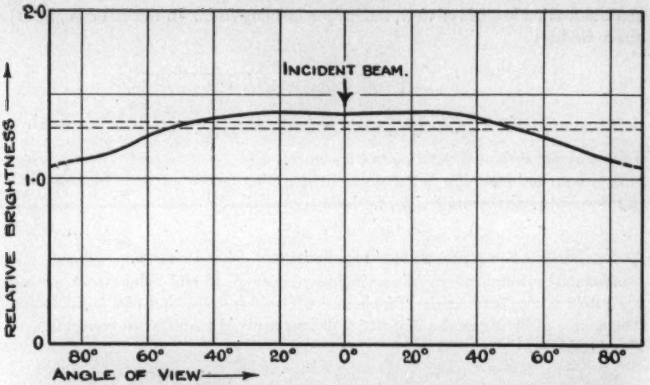


Fig. 1 gives the curve of relative brightness of a magnesium oxide surface in the plane containing the normal, along which the parallel incident beam is directed. The lower broken line represents the brightness of a perfect diffuser reflecting the same total flux as the oxide, while the upper one represents that of a non-absorbing perfect diffuser under the same illumination. It is seen that with a normally incident beam the oxide is equivalent to a non-absorbing perfect diffuser when the angle of view is 45°.

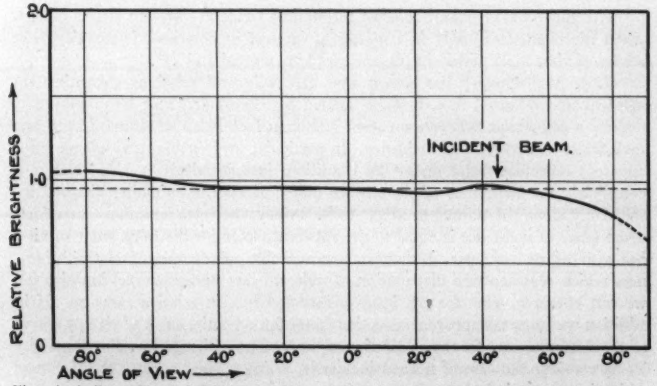


Fig. 2 is similar to Fig. 1, but illustrates the case of 45° incidence. The broken line here represents the brightness of a non-absorbing perfect diffuser under the same illumination. The broken line, corresponding to the lower one in Fig. 1, is omitted as its ordinate is doubtful [see discussion in summary]. The brightness of the oxide is nearly constant over a wide range of angles near the centre of the diagram, and the oxide surface is almost exactly equivalent to a non-absorbing perfect diffuser over this range.

Let $B(\theta)$ be the brightness at angle θ to the normal. Then it is easily shown that the total reflected flux per unit area of surface is

$$\int_0^{\pi/2} B(\theta) \cos\theta \,.\, 2\pi \,.\, d\cos\theta = -\pi \int_0^{\pi/2} B(\theta) \sin 2\theta \, d\theta.$$

In practice we measure $k.B(\theta)$, where k is an illuminometer constant, and can thus obtain

$$-\pi k \int_0^{\pi/2} B(\theta) \sin 2\theta \, d\theta \text{ graphically,}$$

$$= k\phi, \text{ where } \phi = \text{total reflected flux.}$$

Assuming that the apparent reflection factor for normal incidence and angle of view $\theta = \theta_0$ is ρ, we have

$$\pi B(\theta_0) = \rho F, \text{ where } F = \text{incident flux;}$$

whence

$$\frac{k\phi}{\pi k B(\theta_0)} = \frac{\phi}{\rho F} \qquad \ldots\ldots(5).$$

Having measured $k\phi$ and $kB(\theta_0)$ and knowing ρ from the previous calibration, we can calculate ϕ/F, i.e., the total reflection factor for normal incidence. As stated above, this fraction is also equal to the apparent factor for diffuse illumination and normal view. Similar theory holds for any other angle of illumination, except in so far as the polar surface of brightness is no longer symmetrical and the integration giving $k\phi$ may not be so simple as in the case of normal incidence.

Description of Apparatus

The sphere reflectometer was made from a copper shell of 4 inches internal diameter. The shell was spun in two halves and the halves were mounted in flanges giving a butt joint, so that they could be separated for smoking with the oxide. The diameters of the apertures were roughly as follows:

Test aperture	2 inches.
Observing aperture	27/32 inch.
Illuminating aperture	7/32 inch.

The test aperture was cut in a horizontal plane perpendicular to the flange joint, i.e., through the flange, and the centres of the other two lay on radii at 45° to this plane. The construction is best seen from the photographs. The interior surface of the sphere and the inner side of the standard plate (also of copper) were burnished and heavily plated with silver. The illuminating aperture received light from an automobile head-light bulb of rating 12–14 v. 48 w. [gas-filled with single pillar filament] placed at a distance of about 8 cm. on a wooden strip clamped to the mounting of the sphere. The observing eyepiece was of the contrast type, having a comparison surface of matt opal so placed that it could be illuminated by the usual type of comparison lamp travelling on a photometer bench. The Lummer-Brodhun prism was of specially small design to enable the size of the observing aperture to be kept as small as possible. The eyepiece could be detached temporarily and

inclined at an angle when it was desired to bring the artificial sky into the field of view. Although the comparison surface was not on the centre line of the photometer bench, the comparison lamp was never used at a distance short enough to introduce an appreciable cosine error. Whenever readings were taken with the test

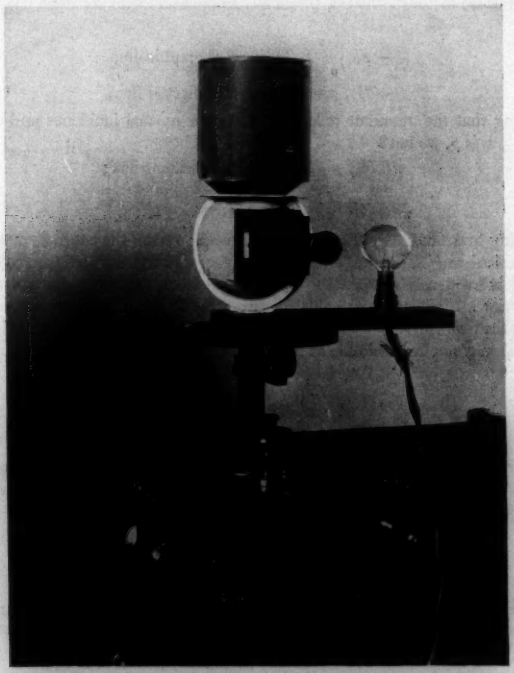


Fig. 3 is reproduced from a photograph showing the reflectometer assembled for measurement of total diffuse reflection factors. The opal comparison surface is clearly seen, and also the screening box surmounting the sphere and covering the test aperture.

aperture open, stray light was excluded by placing a box lined with black velvet over the aperture. The voltages on both lamps were controlled by means of potentiometers. The artificial sky consisted of two coaxial cylinders of equal diameter placed end to end and whitened internally. The upper one contained a small gas-filled projection lamp, while the bottom end of the lower one was closed by a

plate of dense matt opal glass. Between the two cylinders was fixed a thinner piece of matt opal. The diffusion of the light emitted from the lower opal was good, but to avoid error as far as possible, observations of its brightness were always made in the region of 45° with the normal, at which angle a fairly good average value might be expected. Strictly, a polar curve of brightness should have been taken and a calculation made from it of the angle at which the brightness of the opal was equal to that of a perfect diffuser emitting an equal amount of luminous flux. As however the brightness variation was small this was considered unnecessary.

In the case of the measurement of direct reflection factor of a plane magnesium oxide surface by comparison with the sphere, the test aperture of the sphere was treated as though it were a separate standard plate, care of course being taken to

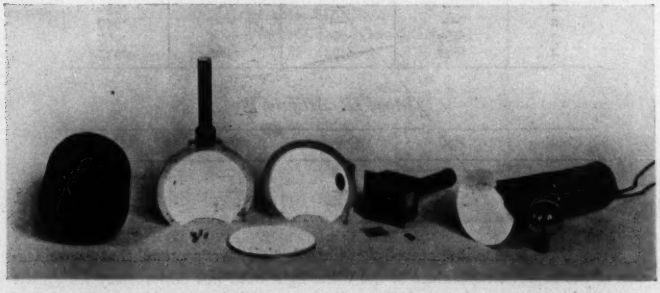


Fig. 4 shows the components of the reflectometer. In order from left to right these are the screening box, the halves of the sphere with the standard plate, the photometer head and eyepiece, the artificial sky and the test lamp for illuminating the interior of the sphere. The square of glass in front of the photometer fits into the square hole in the casing and serves both to protect the prism from the dust and to compensate for the slight green colour of the comparison opal.

see that the other two apertures were in neither the field of view of the illuminometer nor the incident cone of light. When observations were made at normal view and 45° incidence it became clear that if the sphere were removed and the standard plate placed in the plane formerly occupied by the test aperture a slight change in angular setting would involve an error due to change in illumination. To obviate this the sphere was permanently clamped in position, and when readings were to be taken on the standard plate, the plate was fastened in contact with the edges of the test aperture, thus ensuring the *same* angular setting even if it did not happen to be exactly 45°. The reduction in distance from the lamp due to the plate thickness was allowed for in the relative brightness measurements.

These and the brightness measurements for the polar curves were made with the improved form of illuminometer designed at the National Physical Laboratory by Mr A. K. Taylor, and using bridge control of the comparison lamp.

Results

The preliminary figures for the diffuse factor of magnesium oxide have already been given. When it was decided to make a more intensive investigation of the properties of the oxide, its selectivity was also considered and measurements of the diffuse factor were made using red, green and blue glasses in the photometer eyepiece. The following results were obtained for the last few coatings on the bare copper surface:

Method I. Removable plate

Coating	Red	Green	Blue	White
1	—	—	—	·950
2	—	—	—	·949
3	·967	·966	·963	·967
4	·977	·972	·969	·973

Method II. Artificial sky

Coating	Red	Green	Blue	White
1	—	—	—	·946
2	—	—	—	·944
4	·973	·967	·961	·968

From these results it appeared that the selectivity was so small that a specification of the colour temperature of the white light used was unnecessary within the usual limits corresponding to the vacuum and gas-filled types of lamp. In point of fact the light used for the subsequent calibration corresponded to an efficiency lying between these limits, and measurements with the coloured glasses were discontinued. Accurate measurement of the reflection factor was then commenced on a very carefully smoked coating on the silver surface, the procedure being as follows. The total diffuse reflection factor was determined in the reflectometer by the removable plate method. The sphere was then set up on a bench with the test aperture receiving light from a lamp run at steady voltage. The brightness of the interior was compared by means of an illuminometer with that of the standard plate when fixed over the aperture with the oxide surface outwards. After correction for the thickness of the plate, this brightness ratio was used to calculate the apparent reflection factor of the plate by means of the equations (3) and (4) above. Observations were made for normal incidence and 45° angle of view, and for 45° incidence and normal view. A similar comparison was then made against the standard opal plate for normal incidence and 30° view [the conditions of standardisation of the opal]. This gave the apparent factor for these conditions, but subsequent polar curve measurements enabled the result to be modified to give the value for normal incidence and 45° angle of view, which thus afforded a check on the former independent determination. In order to eliminate any possible errors associated with a particular set up

of the apparatus, the whole was dismantled, recoated with oxide and the complete series of measurements repeated.

Constants used in calculations

$c' = \cdot0670$ $\quad c_1 = \cdot0129$

$c'' = \cdot0625$ $\quad a = \cdot9201$

Polar measurements of relative brightness

1. *Normal incidence*

Angle of observation	Mean brightness
0° [Normal]	[Incident beam]
10°	1·40
20°	1·41
30°	1·39
40°	1·36
50°	1·32
60°	1·25
70°	1·17
75°	1·11

2. 45° *incidence.* [*Observations in plane of incidence*]

Angle of observation	Mean brightness
75°	1·03
70°	1·02
60°	0·99
50°	0·97
40°	0·97
30°	0·96
20°	0·94
10°	0·94
0° [Normal]	0·95
10°	0·94
20°	0·92
30°	0·96
35°	0·97
45°	[Incident beam]
55°	0·93
60°	0·93
70°	0·87
75°	0·83

Note. The values in these two tables, being relative values taken on two separate occasions, are not comparable.

From the first table the correction factor to be applied to the apparent factor at 30° view in order to give that at 45° view is 134/139. The apparent factor of the opal, specially standardised for the purpose of this investigation, was 0·805.

Results on first coating

Measurement	Brightness ratio	Result
Total diffuse factor	2·695	$m = 0{\cdot}973_8$
Apparent factor by comparison with sphere	1·794 obs. 1·776 corr.	$m' = 1{\cdot}00_5$ for normal incidence, 45° view
	1·796 obs. 1·771 corr.	$m' = 1{\cdot}00_0$ for 45° incidence, normal view
Apparent factor by comparison with opal	1·294	$m' = 1{\cdot}04_0$ at 30° view $m' = 1{\cdot}00_5$ at 45° view

The apparatus was then dismantled, reassembled and the following results obtained:

Results on second coating

Measurement	Brightness ratio	Result
Total diffuse factor	2·769 by Method I 1·603 ,, II	$m = 0{\cdot}974_9$ $m = 0{\cdot}974$
Apparent factor by comparison with sphere	1·755 obs. 1·737 corr.	$m' = 1{\cdot}00_8$ for normal incidence, 45° view
	1·751 obs. 1·726 corr.	$m' = 0{\cdot}99_8$ for 45° incidence, normal view
Apparent factor by comparison with opal	1·302	$m' = 1{\cdot}05_0$ at 30° view $m' = 1{\cdot}01_0$ at 45° view

Reciprocal calculations

From the polar distribution under normal illumination we have

θ	$\sin 2\theta$	Relative brightness	Product
0°	0	—	0
10°	·342	1·40	0·479
20°	·643	1·41	0·906
30°	·866	1·39	1·204
40°	·985	1·36	1·340
50°	·985	1·31	1·290
60°	·866	1·25	1·083
70°	·643	1·17	0·752
80°	·342	1·01	0·345
90°	0	—	0

From graphical integration of the products we have (see equation (5)) $k\phi = 1{\cdot}31\pi$. Also $kB\,(45°) = 1{\cdot}34$ and $\rho = 1{\cdot}00_5$. Hence the apparent factor under diffused light and normal view is

$$\frac{\phi}{F} = \frac{1{\cdot}00_5 \cdot 1{\cdot}31}{1{\cdot}34} = 0{\cdot}98.$$

In the case of 45° illumination the polar curve is not symmetrical. A sufficiently good approximation to the integral reflected flux may however probably be obtained if we take the mean value at each angle of view (on opposite sides of the normal). We thus construct the following table:

θ	sin 2θ	Relative brightness	Product
0°	0	·95	0
10°	·342	·94	·321
20°	·643	·93	·598
30°	·866	·96	·831
40°	·985	·975	·960
50°	·985	·965	·951
60°	·866	·955	·827
70°	·643	·945	·608
80°	·342	·93	·318
90°	0	—	0

From graphical integration we have $k\phi = 0{\cdot}954\pi$. Also $kB\,(0°) = 0{\cdot}95$ and $\rho = 1{\cdot}00$. Hence the apparent reflection factor under diffused light and 45° angle of view is

$$\frac{\phi}{F} = \frac{1{\cdot}00.0{\cdot}954}{0{\cdot}95} = 1{\cdot}00.$$

[For discussion of this result see summary.]

Summary

The results of the work described above may be summarised thus:

Reflection factors of magnesium oxide

Total factor under diffused light	= 0·974
Apparent factor for normal incidence and 45° view	= 1·00$_8$
Apparent factor for 45° incidence and normal view	= 1·00$_0$
Apparent factor for diffuse incidence and 45° view	= 1·00
Apparent factor for diffuse incidence and normal view	= 0·98

In deducing the apparent factor for diffuse incidence and 45° angle of view by the reciprocal relationship we have made the assumption pointed out above, that an approximation to the total reflected flux for the case of parallel incidence at 45° can be obtained from brightness measurements in the plane of incidence only. The result thus obtained, viz., 100 per cent., is clearly somewhat high since it implies a total factor of 100 per cent. for parallel incidence at 45° also. An accurate evaluation of this factor would necessitate a complete series of measurements in three dimensions of the brightness of the oxide surface. Owing to symmetry, this criticism does not apply to the case of normal incidence.

These figures are in every case the mean of determinations made on two entirely independent coatings of the oxide by two observers, and may be considered accurate, to the number of figures given in this summary with the exception of the result just discussed, which is probably roughly 99 per cent. instead of 100 per cent. Moreover the determination of each constant has been made by two distinct methods, in the case of the diffuse factor by the removable plate and artificial sky, and in the case of the apparent factor under parallel incidence by comparison with the sphere and the standard opal. The good mutual agreement in each case fully justifies the extension which has been made in the use of the sphere reflectometer, especially as regards its adaptation to the measurement of reflection factors under parallel incidence, for which novelty is claimed.

The results themselves bring out the interesting, if not immediately obvious, possibility of the occurrence of apparent reflection factors greater than unity. This apparent anomaly follows the employment of a definition of apparent reflection factor depending on a brightness value measured in a single direction. This value is dependent not merely on the total reflected flux but also upon its distribution. If the distribution is such that a concentration of light occurs in the direction of observation, the brightness in that direction may well be greater than that of a perfect diffuser reflecting 100 per cent. of the incident light. At the same time the brightness in other directions will be low enough to ensure that the total reflected flux shall not be greater than the total incident flux.

The selectivity of a magnesium oxide surface is too small to affect its use for ordinary purposes, and its reproducibility is good provided a polished silver backing is used and care is taken to obtain an adequate thickness of oxide, say not less than 1/3 mm. No special attention has been paid to possible deterioration of such a surface with time, but in the case of several coatings which were kept protected from dust, the diffuse factor did not change by as much as 0·1 per cent. in the course of a fortnight. In all cases the smoking was done with burning magnesium ribbon, the use of protective goggles being found essential to avoid inflammation of the eyes due to the ultra-violet light.

In conclusion the author wishes to thank Mr H. Buckley for valuable criticism and advice, Messrs B. H. Crawford and E. C. Hayward for assistance with the observations, and Mr F. J. C. Brookes for preparing the photographs of the apparatus.

DISCUSSION

Dr J. W. T. Walsh: I should like very much to congratulate Mr Preston on the paper he has presented this evening and on the extremely careful and painstaking work which it describes. On first reading it through, most people who are accustomed to ordinary photometric accuracy will be astonished at the fact that

Mr Preston regards the precision of his results to be one part in a thousand, but I think this is really due to the fact that what he measures is not the reflection factor itself but the absorption factor, so that instead of looking at the figure 0·974 we should really consider the figure 0·026 and clearly an accuracy of one part in twenty-six is not outside the limits for careful work. For measurements of this kind it is possible that we should be able to think more clearly on this particular subject if we always dealt with absorption factors rather than reflection factors.

Mr Preston has pointed out that the slight differences he finds in the reflection factors for light of different colours indicate that his magnesium oxide showed a selectivity in favour of the red, whereas the American workers have found a higher reflection factor in the blue. I do not know whether it is possible that this may be due to different grain sizes of material in the two cases. Has Mr Preston made any study of the grain size of his coatings?

A very important matter in considering the adoption of any surface as a standard of reflection factor is its degree of permanence, and one would like to know whether the magnesium oxide surfaces which Mr Preston prepared change in reflection factor after continuous exposure. I believe Mr Preston has made some preliminary study of this matter, and it would be interesting to hear what conclusions, if any, he has reached.

Mr T. Smith: Mr Preston has carried out an investigation of great interest to myself and my colleagues engaged on optical work in the National Physical Laboratory, and I am confident it will be much appreciated by workers in optics and photometry elsewhere. Some of his results are of particular interest to us, as a surface freshly coated with magnesium oxide has been employed under definite conditions of illumination and viewing as a convenient standard to which the diffuse reflectivity of opaque materials has been referred. The results given in the paper suggest that the figures obtained in this way may be taken without correction as absolute values. My congratulations to Mr Preston on this work are inevitably associated with a feeling of gratitude to him for furnishing us with such a convenient value for this standard.

Dr Walsh has described himself as a "hardened photometrist." The phrase appeals to me as I have frequently regretted the photometrist's persistence in transgressing the laws of the English language by his misuse of technical terms. I have no doubt that almost all scientific workers have occasionally erred in this way, but I am not sure that the photometrist and the illuminating engineer do not deserve to be classed with the wireless enthusiast as the most incorrigible of sinners. I must therefore express my regret, though not my surprise, at realising that Mr Preston has already been corrupted, as is evident from his use of the word "flux." In my opinion the lack of interest of the physicist in certain technical problems is not unconnected with his natural reaction to the language of the technician. It is the fault of the illuminating engineer if his misuse of scientific words leads the scientific worker to disregard him.

I would like to direct Mr Preston's attention to a point in the theory given in

the paper, for I have an instinctive feeling that the first expression developed on p. 18 is wrong. Some terms seem to me to have been disregarded. The magnitudes concerned are doubtless small enough to have no appreciable influence on the numerical results given in the paper, so that the point I wish to discuss is, at the moment, only of academic importance. The relation expressed in words, from which the mathematical developments start, sounds very plausible, and on that account may the more easily escape examination. The difficulty of detecting an error in a verbal statement of this kind will be more readily appreciated when I suggest that it requires amendment by the inclusion of such apparently absurd terms as "radiation escaping through A from A." I should expect a correct formula to involve as a variable only the sum of the solid angles subtended at the centre of the sphere by all the openings. I think we can confirm the accuracy of this view as follows. The shape of the openings and their relative angular positions are not factors in the result. We may therefore modify the shape, but not the area, of an opening without altering the rate at which energy is lost. We may also bring two openings into any relative positions we choose. Let their shapes be so chosen that their contours are exactly complementary to one another over a suitable portion of their boundaries, and let the two be brought together with the corresponding portions of the boundary fitting together. We then obtain a single opening. This opening can be split up into two by the reverse of the process just described, the ratio of the areas of the two new portions differing from the old ratio. The value of this ratio is physically without meaning before the areas are separated, and it is admitted that subsequent separation is without effect on the quantity concerned. It follows at once that we cannot have a term $A_s B_s$ in the formula, which changes in value with the ratio A_s/B_s, without having also terms in A_s^2 and B_s^2 to make the whole function independent of this ratio. Instead of the formula given we must therefore substitute a formula involving only the single effective variable $A_s + B_s + C_s + \ldots$. This argument shows that, despite the paradoxical form in which the idea presents itself, the radiation from A which escapes through A is something real which has to be taken into account. Knowing that the correct formula is a function of this particular variable, it should not be a difficult problem to determine the function itself. The exact expression is not unlikely to be an algebraic fraction. As a guess, the radiation retained in the sphere may be measured by

$$\frac{\theta}{1 - R\theta},$$

where θ is the fraction of the whole surface of the sphere not removed, and R is the ideal reflection factor. On this scale the radiation without any openings is measured by $(1 - R)^{-1}$, and the fractional loss of radiation due to openings amounting to the fraction Σ of the whole surface would be

$$\frac{\Sigma}{1 - R + R\Sigma}.$$

[Since hazarding this guess I have proved the more general result that the ratio

of the diffused light falling on every point of the surface of the sphere to the initially incident light is equal to

$$\frac{\Sigma}{1-P},$$

where

$$\Sigma = S_a\alpha + S_b\beta + S_c\gamma + \ldots, \quad P = R_a\alpha + R_b\beta + R_c\gamma + \ldots,$$

α being the portion of the surface of the sphere which has S_a as its reflection coefficient for diffused light and R_a its coefficient for initially incident light. It is of course assumed that the diffusion conforms to the ideal law.]

Mr J. S. Preston (partly communicated): Dr Walsh has drawn attention to the accuracy claimed for the results on the diffuse reflection factor of the oxide, and I agree that the reason for this claim is the fact that the quantity measured is really the absorption. It is possible that the theoretical treatment would be shortened by using expressions representing light absorbed rather than light reflected, though I have not investigated this point.

As regards the grain size of the oxide, I have no information upon the coatings actually employed, but the difference in selectivity between different coatings may very possibly be connected with a difference in grain size. This view is supported by the fact that the American workers have mainly used magnesium shavings burnt in bulk, somewhat in the manner of a photographer's flashlight, whereas the coatings used for the investigation just described were obtained with slowly burning ribbon.

The permanence of the surface has not yet been fully investigated. There is, however, a certain amount of evidence for a slight change in reflection factor due to gradual change of oxide to carbonate. The change is almost inappreciable and other causes are not yet definitely ruled out of court. It is hoped, however, that further information on this point will shortly be available.

Mr T. Smith objects to the photometrist's misuse of the English language. No photometrist would deny the charge; in fact I think most photometrists share, to some extent at least, the regret expressed by Mr T. Smith. I agree that the lack of interest of the physicist in some technical problems is often connected with his reaction to the language of the technician. On the other hand, when the purely physical aspect of a branch of science makes contact, almost at every point, with the technical, a certain broadness in the use of terms is not only unavoidable but probably beneficial in securing a mutual understanding. Photometry has certainly suffered from this temporary state of affairs in recent years and the effects are still evident. If excuse were necessary, I think the best would be that lack of precision in the use of scientific terms often does denote a healthy co-operation between the pure and applied sections of a branch of science.

With regard to the derivation of the expression on p. 18, I hope the following alternative derivation will be convincing and also reveal the cause of difficulties raised by the somewhat intuitive reasoning first employed.

Let A, B represent the plane areas of two circular apertures in the sphere, A_s, B_s their spherical areas. Let b be the brightness of the sphere interior and for convenience let its radius be unity.

The flux lost through A is simply shown to be

$$\tfrac{1}{4}bA_s \times \text{area of sphere remaining}$$

$$= \frac{bA_s}{4}[4\pi - A_s - B_s].$$

Thus total flux lost through A and B is

$$\frac{b}{4}[4\pi(A_s + B_s) - 2A_sB_s - A_s^2 - B_s^2].$$

Now A and A_s are connected by the equations

$$A = \pi \sin^2\theta,$$

$$A_s = 2\pi(1 - \cos\theta),$$

and similarly for B and B_s,

$$B = \pi \sin^2\phi,$$

$$B_s = 2\pi(1 - \cos\phi).$$

Using these relations we may re-write the loss of flux thus:

$$\frac{b}{4}[8\pi^2(2 - \cos\theta - \cos\phi) - 4\pi^2(1 - \cos\theta)^2 - 4\pi^2(1 - \cos\phi)^2 - 2A_sB_s]$$

$$= \frac{b}{4}[8\pi^2 - 4\pi^2\cos^2\theta - 4\pi^2\cos^2\phi - 2A_sB_s]$$

$$= b\left[\pi^2\sin^2\theta + \pi^2\sin^2\phi - \frac{A_sB_s}{2}\right]$$

$$= \pi b\left[A + B - \frac{2A_sB_s}{4\pi}\right],$$

which is the expression derived in the paper putting $R = 1$. It is clear that in this form, utilising both plane and spherical areas, the expression may be extended to cover any number of circular apertures.

With regard to the formula $\dfrac{\theta}{1 - R\theta}$ for the radiation retained in a part sphere, I would refer Mr Smith to the paper by Benford in the *General Electric Review*, January 1920. Here the brightness of the walls of a part sphere is shown to be proportional to $\dfrac{R^2}{1 - R\theta}$, provided radiation is supplied at a constant rate. This latter formula, although strictly applicable to a removable cap of spherical, not plane contour, gives results for the magnesium oxide which agree to about 0·01 per cent. with those given above, using the same brightness ratios. The formula $\dfrac{\theta}{1 - R\theta}$, although in fairly close agreement under the conditions of the actual experiments, viz. both θ and R near unity, shows a departure of 0·5 per cent. The difference between the two formulae becomes evident if we make R zero. In this

case, since the sphere is perfectly black, no luminous radiation is retained, whereas the formula $\frac{\theta}{1 - R\theta}$ reduces to θ instead of to zero.

The derivation of the last of Mr Smith's formulae, namely $\frac{\Sigma}{1 - P}$, suggests however that he is thinking of a point source of light inside the sphere, and indeed this is possibly the cause of the discrepancy between the other expressions just discussed. I should like to point out that in the type of reflectometer under discussion the incident radiation is constant, in the form of a narrow beam, irrespective of the size of any apertures in the sphere, and that the brightness measurements represented the diffuse radiation only, no readings being taken on the part of the sphere receiving the initially incident light.

A STEREO-FIXATION APPARATUS FOR USE WITH BJERRUM SCREEN

By E. F. FINCHAM

(Northampton Polytechnic Institute, London)

MS. received, 14th November, 1929. Read and discussed, 14th November, 1929.

ABSTRACT. The fact that binocular fixation is more constant than the monocular method makes the stereoscopic campimeter more accurate than the monocular instrument. In addition, this method allows records to be obtained of central blind areas. In previously existing stereo-campimeters the Brewster type of stereoscope has been employed. These instruments have the disadvantage that the test is made upon a small screen and that the field is limited on the nasal side by the usual septum between the two fields. This paper describes a new apparatus giving stereoscopic fixation for use with a large Bjerrum screen and allows a field of examination of 35° in all directions from the centre. The apparatus consists of a simple optical system which presents to the eye not under test a magnified image of a small design which is the stereoscopic counterpart of a design seen by the other eye at the centre of the Bjerrum screen. The images obtained by the two eyes are fused, giving stereoscopic fixation. Means of adjustment for muscle imbalance is provided. The exploration of the field of vision is carried out in the usual manner. The paper contains a copy of a record made with the new apparatus of a central scotoma in a case of old atrophy of the macula.

Introduction

THE purpose of this apparatus is to secure and maintain steady fixation of the eye while examination is being made for blind areas in the field of vision.

The usual method of making this examination is to occlude the eye not being tested and to instruct the subject to keep his vision fixed upon a small central mark on a black screen (Bjerrum screen). A small white or coloured object is then moved about the screen. By marking the points of disappearance of the object, the boundaries and angular size of the blind area may be recorded as well as its position relative to the point of fixation.

This original method is open to some objections. The monocular fixation of a simple object such as a small mark on a black screen cannot be depended upon with certainty. Moreover in cases of scotoma involving the fovea, central fixation is lost and hence the method described cannot be applied. As the accurate mapping of the scotoma in such a case may be of some importance in diagnosis another method of fixing the eye must be resorted to.

Lloyd* has made use of a stereoscope of the Brewster type for this purpose. His instrument has been designed to give a large field of view and has in the

* The ordinary stereoscope was first used in campimetry by Haitz in 1904. Lloyd introduced a specially designed instrument, manufactured by the Bausch and Lomb Optical Company, in which the field is as large as can be obtained with the Brewster type of stereoscope (*New York Med. Journ.* Dec. 1920).

position of the slide a black screen bearing two diagrams in grey lines which the eyes fuse to form a stereoscopic image. The examination of the field for each eye is carried out in the usual manner.

It has been found that this binocular fixation of a stereoscopic field is much more constant than the simple monocular method. In cases of a central scotoma of recent origin the peripheral parts of the diagram seen with the affected eye are mentally combined with the image in the other eye, and thus fixation is maintained.

Description of the New Apparatus

As the measurements of the field of vision and of blind areas within it are angular values the records made upon a flat screen will be tangents. Therefore the size and accuracy of the records will be increased by increasing the distance of the screen from the eye. The Brewster type of stereoscope has the disadvantages that the distance of the screen is small and that the field of each eye is restricted on the nasal side.

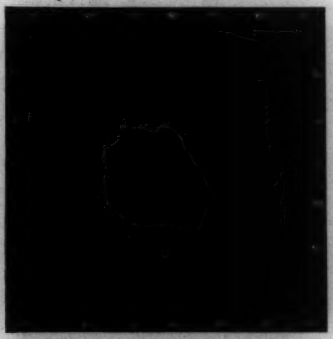


Fig. 1. The irregular white line is the map of a scotoma which was recorded recently on this apparatus in a case of old atrophy of the macula. The small spot below the central area is caused by a spot of pigment immediately above the macula. The test object employed was a 1 mm. disc at a distance of 1 metre. The radii of the circles on the chart subtend at the eye the angles of 1, 2, 4 and 6 degrees respectively.

In the new apparatus the principle of stereoscopic fixation has been applied for use with a large Bjerrum screen (5 ft. square) employed at a distance of one metre, and is so arranged that the available field is 35° in all directions from the centre.

The arrangement of the apparatus is as follows: at the centre of the Bjerrum screen is hung a black card about 1 foot square which bears a design (Fig. 1) (printed) upon it in grey ink. This is viewed direct by the eye under test. The

patient's head is held in a suitable rest and the eye-cup of the instrument (Fig. 2) is brought in front of the other eye.

To this eye is presented by means of a mirror and suitable lens, a reversed replica of the design upon the screen. The size and distance of the small replica of the target and the power of the lens are such that the image appears to be at the same distance as the screen and of the same size as the target upon the card. The binocular images can therefore be fused.

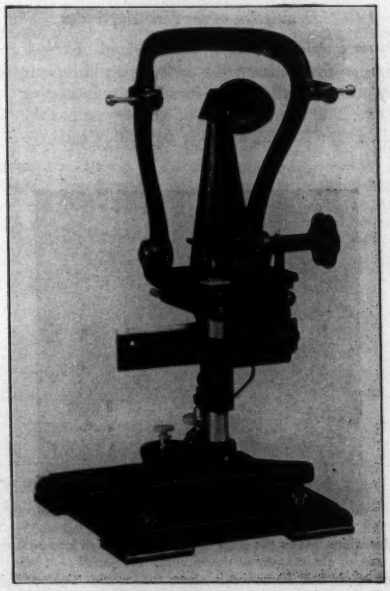


Fig. 2.

It will be seen in Fig. 1 that the squares are decentred laterally with regard to the central spot and the circles. The small target in the apparatus is reproduced from this by photography and is inverted so that the decentration is to the opposite side to that of the large target upon the screen. The effect of this condition is that the binocularly fused image shows stereoscopic relief, either the squares or circles "standing out" according to which has the least separation in the two images.

In order to allow fusion in cases of muscular imbalance a horizontal or vertical

displacement of one image can be produced by rotating the stand of the instrument upon the table or tilting it backward or forward with a levelling screw. If the patient sees two images of the target he is instructed to make the necessary adjustment to bring them together when he will be able to describe the stereoscopic appearance.

The apparatus is provided with an adjustable chin-rest so that the eye may be brought to the required height. The part carrying the optical system is capable of a backward and forward and lateral movement in order to bring the eye-cup up to the eye. The eye-cup is large enough to occlude the Bjerrum screen completely from the eye which is not being tested, and also to allow correcting lenses to be worn in cases of refractive error.

The part of the apparatus carrying the small picture of the target and the optical system may be moved across to bring it in front of either eye so that that eye is made to assist in fixation while the field of the uncovered eye is being tested. The effect of moving the apparatus from before one eye to the other is to alter the stereoscopic relief from either circles or squares appearing to be nearer.

Both the large screen and the card which hangs upon it are marked with radial lines at every 15° and the screen has also circular divisions to assist in making records. These lines are sufficiently faint to be invisible to the patient. The surface of both screen and card have been prepared to allow records to be made upon them with Conté chalk which can be removed with a damp sponge. Permanent records may be made by transcribing the results on to a small paper chart bearing the same divisions as the large screen.

In conclusion, I should like to say that much of the mechanical design and the production of the apparatus is due to Mr W. Green, of the Elliott Optical Company, London.

DISCUSSION

Mr T. Smith: I have found that it is easier to maintain stereoscopic fusion when looking at photographs of natural objects than when a geometrical design is used. If this is a general experience there would be some advantage in adopting an attractive photographic object for fixation rather than the centre of a geometrical pattern.

Mr E. F. Fincham: In reply to Mr Smith I must say that my own experience agrees with his. The target, however, has to serve the purpose not only of providing a stereoscopic object for fixation but also a chart having definite angular divisions upon which the boundaries of blind areas are recorded. Moreover, it must be only faintly seen and of such a nature that it will not detract the subject's attention from the small moving test object. I fear that these conditions would be difficult to obtain if a photograph of an actual scene or object were used. No difficulty has been experienced with the present target with subjects having apparently normal stereoscopic vision.

EXHIBIT OF THE BUSCH OPTOMETER

By the EMIL BUSCH OPTICAL Co., Ltd.

Exhibited, 14th November, 1929.

This is an instrument for measuring objectively the errors of an ametropic eye. By it one can measure in diopters the hypermetropia or myopia of a symmetrical eye or the ametropia in the two meridians of an astigmatic eye together with the directions of the two meridians corresponding with the two powers of the eye.

The patient looks with one eye into the exit pupil of the instrument whilst the observer looks into the eyepiece and focusses on the patient's macula the image of the straight filament of a small electric bulb. The focussing head moves a scale relatively to a fixed pointer so that when the side edges of the filament are sharp the ametropia may be read off at once.

Optical System

Fig. 1 shows diagrammatically a vertical section of the essential parts of the instrument and of the patient's eye when the ametropia of the horizontal meridian is being measured.

We will trace the path of the light from the single filament lamp to the retina and then back to the eyepiece.

The right-angled reflecting prism forms a vertical image of the filament in the first focal plane of objective 1, so that the light from a point of the image after passing objective 1 is parallel, and after passing objective 2 forms an image at its second focal plane which, in the initial position when the pointer is at zero on the scale, coincides with the first focal plane of objective 3. After passing objective 3 the light again becomes parallel and enters the upper of a pair of rhombus prisms. After two reflections it emerges as a parallel beam and enters the patient's pupil. If the eye is emmetropic a vertical image of the filament will be focussed on the macula and will illuminate the fundus.

Starting now from left to right and considering the image on the macula as the source of light, the rays from a point of the source on emergence from the eye will become parallel, and after passing through the lower rhombus prism will be still parallel and will form an image in the second focal plane of objective 4, identical with 3. The adjacent focal planes of objectives 4 and 5 being coincident, the objective 5, identical with 2, makes the light again parallel, and objective 6, identical with 1, forms an image in the first focal plane of the eyepiece where the cross-wires are situated. The pencil then falls on the lenses of the eyepiece and emerges parallel

into the eye of the observer who will thus see a sharp image of the filament on the fundus coincident with one of his cross-wires.

Between objective 6 and the eyepiece a rhombus prism shifts the pencil upwards so as to bring the optic axis of the eyepiece in line with the axis of best vision of the patient's eye.

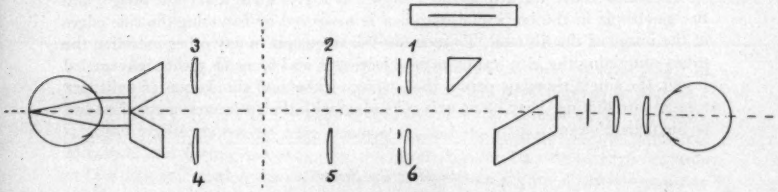


Fig. 1. The Optical System.

Symmetrical Ametropic Eye

The retina may be regarded as a screen on which the image of the lamp filament is projected. When the eye is ametropic the image will not be sharp. If the eye is hypermetropic the image will be projected beyond the retina, and in order to bring it on to the retina, when the objective 3 and the rhombus prisms remain at rest, it will be necessary to move the objective 2 to the right, so that its second focal plane will no longer coincide with the first focal plane of objective 3. If the patient's eye is myopic the movement must be to the left as the image is projected short of the retina. The parts of the instrument shown to the right of objectives 3 and 4, including the diopter scale, are carried on a separate slide and are moved bodily to right or left when the focussing head is turned.

It may readily be shown that if d is the distance the slide is moved to the right or left of the initial position, and if D_3 is the power of objective 3 and D_1 the principal point refraction of the patient's eye, we have the simple relation

$$d = \frac{D_1}{D_3^2},$$

which shows that the motion of the slide is directly proportional to the principal point refraction of the patient's eye and that the divisions of the scale are uniform.

This formula, however, is only valid when the adjacent principal points of the eye and of objective 3 are separated by a distance equal to the focal length of objective 3. In the Busch instrument this condition is not fulfilled, and the above simple formula does not apply and the divisions of the diopter scale are not therefore uniform. What is measured and indicated on the scale is the vertex power of the proper correcting lens when its vertex is placed at the usual distance of 12 mm. in front of the vertex of the patient's cornea. In the instrument itself the vertex of

the cornea should be placed at a distance of 15 mm. from the junction of the two identical rhombus prisms.

The scale of each instrument is calibrated at several points by experiment against known refractions.

Astigmatic Eyes

As stated above the arrangement shown in Fig. 1 gives a vertical image, and the ametropia in the horizontal meridian is measured by focussing the side edges of the image of the filament. To measure the ametropia in any other meridian the tubes containing the identical pairs of objectives 1 and 6, 2 and 5, the right-angled prism, the single rhombus prism, the eyepiece lenses and the source of light, are rotated into that meridian on an axis coincident with the optic axes of eyepiece and of the patient's eye.

Reading the Scales

The position of the pointer which indicates the ametropia on the diopter scale is read by the observer who, after focussing, moves his head aside and looks through a magnifier and prism arrangement, the scale being illuminated by a small electric bulb with a push-button. At the same time he sees in the same field a portion of the angle scale engraved on a ring carried by the rotating part of the apparatus so that the angle is read against a fixed pointer. The diopter scale has a range from + 20 to − 20 and is divided into quarter diopters, minus values red and plus values black.

Patient's Fixation Point

In the first focal plane of objective 3 is placed a window diaphragm, with two cross-bars on one of which on the side towards the patient is placed a small blue or green spot which forms the fixation point. As this spot is in the focal plane of objective 3, the rays from it which enter the patient's eye are parallel so that it will appear to him to be at infinity, and accordingly he will relax his accommodation.

The patient thus sees in his field of view, Fig. 2, a bright image of the lamp filament crossed by two black bars on one of which is the blue spot. The eye not being measured is blocked by an opal disc carried on a ring capable of rotation to enable either eye to be obscured. A slide carrying the whole of the apparatus shown in Fig. 1 can be moved longitudinally towards or from the eye, and laterally so that the rhombus prisms can be set opposite either eye.

The Observer's Field of View

The observer sees, Fig. 3, an image of the lamp filament divided into three parts by the two cross-bars of the window. When properly focussed the side edges of the filament are sharp, also the top and bottom edges of the cross-bars which should be at right angles to the length of the filament.

When the eye being examined is astigmatic and the image of the filament does not coincide with one of the principal planes, the edges of the cross-bars will not be at right angles to the length of the filament. By turning the rotatable part until they are at right angles we can set the apparatus in position to measure the ametropia in one of the two meridians and note the direction on the angle scale and check the directions by seeing that the difference of the readings in the two positions is 90 degrees.

Stops

Freedom from glare and from corneal reflections is secured by arranging two semicircular stops in the focal planes of objectives 2 and 5 and close to objectives 1 and 6. The upper stop in Fig. 1 is arranged just below the common axes of objectives 1, 2 and 3 with its curved edge downwards and the diameter of the semicircle horizontal and cutting the axes, and the lower stop has its curved edge upwards and its diameter cutting the common axes of objectives 4, 5 and 6. In consequence of this arrangement the pencil of light projected on to the cornea will be semicircular in section with its curved edge upwards as shown in Fig. 4. Similarly, if

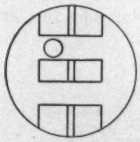


FIG. II
Seen by the patient.

FIG. III
Seen by the observer.

FIG. IV
Aligning the Instrument.

the lower stop be projected on to the cornea we should get a semicircle with the curved edge downwards. It follows, therefore, that all the light coming from the lamp is incident on the top half of the eye, and all the light that reaches the observer from the retina must come from the lower half of the eye. It also follows that light reflected from the cornea cannot enter the eyepiece and the observer's eye.

Near the upper of the semicircular stops is an iris diaphragm to limit the amount of light entering the patient's eye.

Adjusting the Distance of the Cornea

On each side of the instrument is arranged a finder consisting of (1) a lens, (2) a prism having black vertical lines on two of its faces. These two lines or sights determine the direction of a line inclined to and cutting the combined axes of eyepiece and patient's eye at the point where the vertex of the cornea is to be when 15 mm. from the junction of the rhombus prism. In adjusting the instrument after the patient's head is fixed in place on the chin and forehead rests, the observer

looks into the finder lens. He will see a semicircle of light on the patient's eye, see Fig. 4, crossed by two lines. He will then slightly move his eye laterally till the lines coincide. When this is done he will move the slide which carries the whole of the apparatus shown in Fig. 1 in a longitudinal direction towards or away from the patient's eye until he sees that the coincident lines bisect the pupil. When this is done he will know that the centre of the pupil is on the axis of the eyepiece and at a distance of 15 mm. away from the rhombus prisms.

Binocular Fixation

In place of the opal disc a small tilted mirror is arranged to reflect into the eye not being measured an image of a small blue bulb placed on the ceiling of the room some 6 or 8 metres away. Whilst one eye fixes the blue spot in the instrument the other fixes the blue bulb. As both eyes then fix distant objects the two images will be combined stereoscopically and perfect relaxation of accommodation will be secured.

In a modified form of the fixation mark, instead of having a pair of cross-bars, a small grating will be used.

EXHIBIT OF A NEW STEREOSCOPIC HAND OPHTHALMOSCOPE, DESIGNED BY PROF. THORNER

By the EMIL BUSCH OPTICAL Co., Ltd.

Exhibited, 14th November, 1929.

In this instrument a real image of the fundus to be examined is formed by a concave mirror instead of by a lens, as usual, and this image is reflected by a plane mirror into the two objectives of a small double terrestrial telescope provided with a binocular eyepiece.

The two pupils of the observer lie in the exit pupils of the eyepiece and are consequently imaged in the entrance pupils of the telescopes where a double circular diaphragm is placed. Images of these diaphragms are formed by the concave mirror in the upper half of the patient's pupil. It follows that the observer's pupils are conjugate with two small areas of the patient's pupil.

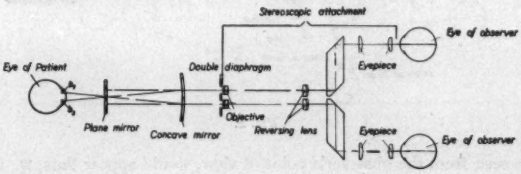


Fig. 1.

An image of a source of light is formed by a condenser on a reflector placed just above the double diaphragm, and in consequence an image of the lamp filament is formed by the concave mirror on the lower part of the patient's pupil and thus illuminates the fundus.

Light reflected from the fundus and passing through one of the images of the diaphragm on the pupil of the patient enters one eye of the observer, and light reflected through the other image enters the other eye of the observer who thus sees two slightly different pictures of the fundus from two separated points of view and combines them stereoscopically.

Since the image of the source of light on the patient's pupil is quite separate from the two images of the diaphragms, it is clear that light from the lamp reflected from the cornea of the patient cannot enter the observer's eyes. Stray light reflected from the patient's cornea may, however, reach the observer's eyes.

The Optical System

This is shown in vertical section and plan. Referring to the dotted lines in the vertical section we see that light from the lamp is focussed on the reflector (not shown), near the diaphragm, and is reflected by the plane and concave mirrors and focussed on the lower part of the patient's pupil. Referring to the full lines in both views, it may be seen that light reflected from the fundus passing through or near the point b_1 is reflected by the concave and plane mirrors and enters the telescope on the observer's left hand, and so enters his left eye. Similarly light reflected from the fundus through or near the point b_2 enters the right eye.

The function of the two prisms is to reduce the distance between the observer's pupils so that they can both fit into one of the patient's dilated pupils.

Suppose a small capital letter R to be printed on the fundus of the patient's eye which we will assume to be emmetropic. The optical system of the eye will form at infinity a real inverted and reversed image of this letter which may be represented thus, Я. The rays emerging from the eye will be intercepted by the concave mirror, which will reflect them and form a real image in space which, if it

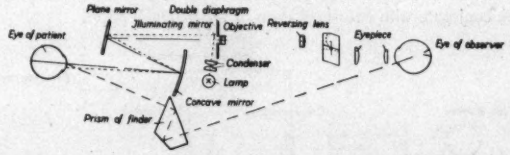


Fig. 2.

could be seen from the observer's point of view, would appear thus, Я. After reflection by the plane mirror the image which could then be seen by the observer would still appear thus, Я.

Now consider the rays passing though one of the telescopes. The objective, the reversing lens and the eye-lenses form a terrestrial telescope which neither inverts the object up and down nor reverses it right and left. The two parallel reflecting faces of the prism act like a simple periscope and neither invert nor reverse, hence the observer sees the capital letter thus, Я. It is both inverted up and down and reversed right and left.

The distance of the plane mirror from the concave mirror is nearly equal to the focal length, that is, about half the radius of the concave mirror. The prisms make the optic axes of the telescopes and the eyepieces parallel. The observer therefore looks with both eyes into one of the patient's eyes, something like a policeman peering through a window pane into a dark room which he illuminates with his bull's eye, or like an inhabitant of Lilliput peeping into one of the supine Gulliver's eyes.

In the elevation is shown a prism which enables the observer to adjust the instrument so that the image of the straight filament of the lamp appears on the lower part of the patient's pupil. The patient will see, in the instrument, a red and green light, and when he fixes the green light with his right eye or the red light with his left his pupils will be exactly in the middle of the observer's field of view.

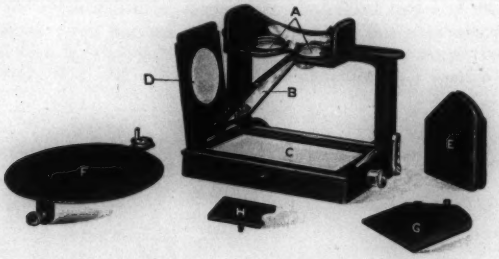
The ametropic of the observer may be corrected by adjusting the eye-lenses separately and that of the patient by turning a milled head which moves the reversing lenses or by rotating a disc carrying a number of small lenses one of which can be turned into position in the telescope and so alters its power.

EXHIBIT OF THE MADDOX CHEIROSCOPE

By C. W. DIXEY & SONS

Exhibited, 12th December, 1929.

The cheiroscope is designed to aid in the training of squinters, and adopts for the first time the principle of utilising both hand and eye in the education of the amblyopic eye, thereby increasing the patient's power of concentration and persistence.



The instrument consists mainly of two spherical lenses (*A* in diagram), a movable mirror (*B*), a drawing surface (*C*), and a picture holder (*D*). In cases where the eyes are comparatively free from squint, the mirror is used either (1) "covered" or (2) "absent." For pronounced squint the mirror is set (3) "oblique," and for extremely convergent squint (4) "vertical."

(1) *The covered mirror.* The mirror should be placed in the vertical position, and then obscured by the double screen (*E*). A small picture or toy can then be placed on one side of the partition, and the patient directed to draw over the image which will be seen on the other side of the partition.

There are many possible variations of this exercise—for example the patient, holding a net on one side of the partition, may try to catch a "butterfly" moved about by the trainer on the drawing surface on the other side of the partition.

To cultivate fusion-power, two similar objects may be placed on the drawing surface, one on either side of the partition, and moved slowly apart.

(2) *The absent mirror.* The mirror is removed from its bearings after withdrawing the T-pin. Beads, counters or other small objects can then be scattered over the whole of the drawing surface, and the patient directed to make designs

by arranging the objects with forceps. The patient will see a second image which he will be unable to move. A rotating table (*F*)—supplied if desired as an accessory —can be fitted over the drawing surface. Small toys placed upon it near its centre will, when the table is rotated, appear to one eye to be moving clockwise, and to the other counter-clockwise. This is an excellent stimulant for both eyes.

(3) *The oblique mirror*. The mirror is set at an angle of 40° from the vertical, in which position it is held by a catch. The whole of the drawing surface is now left free for observation by one eye, while the other will see a reflected image of objects placed in the picture holder (*D*), which either with its rotating frame or cigarette card holder (*H*) is clamped into position at that side of the instrument which is nearer to the free end of the mirror. The patient should draw over the image seen upon the drawing surface.

(4) *The vertical mirror*. The mirror is set vertically and both sides left uncovered. As each surface gives a laterally inverted reflection, the two images are accordant and can be fused. For lesser squints only one surface of the mirror is used, the other being covered by the single screen (*G*).

N.B. If possible the training should be preceded by prolonged occlusion of the fixing eye.

EXHIBIT OF THE DUOCHROME REFRACTION TEST

By Mr CLIFFORD BROWN

Exhibited, 12th December, 1929.

This test utilises the chromatism of the eye for the detection of refractive errors, the object being to provide a more sensitive test than has hitherto been available.

The test letters are exposed in two rows, one on a red and the other on a green illuminated ground. The colours used have their wave-lengths equally spaced from the central sodium line. In the normal eye, therefore, the green will be focussed in front of the retina, and the red the same distance behind. Both images will be slightly blurred, but *equally* so.

Any departure from this condition will cause one image to be brought nearer the retina and made sharper, and the other carried exactly the same distance away, and consequently more blurred.

On this test, therefore, a 1/8 *D* lens placed before an eye is equivalent in effect to a 1/4 *D* lens used on a white light test, since it increases definition of one colour and decreases the other.

Since a simultaneous comparison is offered the effect is more than doubled, and is further helped by the approximately monochromatic light used.

Thus, in the smallest errors, the myopic eye will see the red letters better and the hypermetropic eye the green letters.

A revolving block of red and green lines enables the refraction to be determined in any one meridian, and by this means small astigmatic errors are exposed.

As the hypermetrope will see the green better, the eye will not readily make an effort to accommodate to the red at the expense of the green, and this test, therefore, tends to relax the accommodation.

A further point of importance is that the test provides a definite turning point, after which no more plus can be added without bringing the red into sharper focus. The reason for this, of course, lies in the fact that a *balance* is required. *Equality* of definition is sought for, and not best definition as in tests employing white light; there is, consequently, only one lens which will secure this equality.

Fig. 1.

REVIEWS

Penrose's Annual. The Process Year Book and Review of the Graphic Arts. Vol. XXXII. Edited by WM. GAMBLE, F.R.P.S., F.O.S. Pp. xvi + 168. (London: Percy Lund Humphries and Co., Ltd., 1930.) Price 8*s.* net.

The present volume contains the following articles: "The Editor's Review," "The Oxford University Press," "The English and American Fifty Books of 1928," "The Standardisation of Trichromatic Inks," "Printers can get More and Better Business Through Art," "Consistency of 'Motif' in Advertising Presentation," "How to Transform a Photographic Negative into an Image in Colours," "Fine Edition Books in France," "The Revival of Wood-Engraving," "The Stencil Process in France and England," "Framed Print as Decoration," "Method of Photographing Paintings in Confined Spaces," "Photogravure," "Standardising Three-colour Inks from the Printer's Point of View," "George Baxter: Pioneer Colour Printer," "The Half-tone Process: Improvements in Technique," "The Amateur Cinematographer," "Enlarging from Sub-Standard Film," "The Process Engraver's Lens Stops," "Printing Margins," "Advertising the Empire's Products," "Pantone Developments," "The Kodacolor Process Explained," "Colour in Print," "The Importance of Colour in Advertising," "Historic Half-tone Stops," "Eliminating the Cross Hatch in Making a Half-tone from a Half-tone," "A Note on some Types used in Current Advertisements," "British Printers and Foreign Advertising," "Elements of New German Typography," "Screen Negatives on Dry Gelatine Process Plates," "A Simultaneous Exposure Three-colour Camera," "Further Experience with the Erwin Grain Screen," and "Repairing Holes in Intaglio Cylinders." The volume contains plates illustrating the articles, in addition to a supplement of numerous illustrations representative of process work during the year 1929.

La Topographie sans Topographes. Traité de Photogrammetrie. By Commandant F. OLLIVIER. Pp. xviii + 301; 158 Figs. (Paris: Éditions de la "Revue d'Optique théorique et instrumentale.")

This book deals with the development of surveying from photographs exposed on vertical planes on the ground, the treatment of inclined ground photographs and air photographs being deferred to a second volume. The treatment is mainly historical and contains certain errors and omissions. Pulfrich, for instance, is described as the originator of stereophotogrammetry, whereas Fourcade is more entitled to that distinction. Full references in support of this statement will be found in a paper by H. G. Fourcade entitled "On Instruments and Methods for Stereoscopic Surveying" (*Transactions of the Royal Society of South Africa*, **14**, Part 1, 1926). This paper contains a detailed description of the construction and use of a Stereocomparator, which is considerably superior to the better-known Pulfrich-Zeiss instrument and is far more worthy of study.

The author's views on stereoscopic vision do not appear to have progressed much beyond Helmholtz's Theory of Corresponding Points. Apparent exceptions to this theory are explained by the substitution of corresponding "zones" for "points," and the suggestion that one of the double images is suppressed, or "neutralised," by visual or psychological processes. In plain English, the theory does not fit all the observed facts, and is therefore no longer a valid theory. The influence of the limited field of distinct

vision, which has everything to do with this question, is not discussed. Yet it is surely idle to investigate in detail whether an object is seen single or double under certain conditions of fixation when the object cannot be seen distinctly at all.

The question of stereoscopic distortion arising from incorrect placing of the photographs under stereoscopic examination is treated analytically. This has been done before, notably by Bolton in "Stereoscopic Distortion" [*Photographic Journal*, **43**, No. 4, April 1903 (107)]. Having arrived at the geometrical form of the reconstructed space image in various cases, the author does not discuss sufficiently clearly how much of the resulting distortion can be *appreciated* under the restrictions imposed by insensitivity to impressions of absolute convergence or accommodation. He does, however, arrive at the correct conclusion that this whole question is of little practical moment as regards the applications of the subject.

Within its self-imposed limits of subject matter, and subject to a certain historical bias, the book is a complete and clear description of the methods and instruments used in this branch of surveying, together with a statement of the principles on which these methods rest and examples of results obtained by them. It will repay study by anyone interested in this subject.

M. H.

Proceedings of the Seventh International Congress of Photography, 1928. Edited by W. Clark, T. Slater Price and B. V. Storr. Pp. xiii + 572; 232 Figs. (Cambridge: W. Heffer & Sons, Limited, 1929.) Price 25*s*. net.

One of the most valuable results of the Congress of Photography is the publication of this very complete record of the proceedings. It has been distributed amongst the members of the Congress, but is now made available for a much wider circle of readers who, though not able to attend, are none the less keenly interested in the problems that were discussed. Photography has now many ramifications, but every branch of it seems to have been dealt with very thoroughly in the papers read at the Congress. Even the art side is not neglected, for the report opens with a readable paper by Mr F. C. Tilney. A very large section, running through 258 pages, is devoted to the chemistry of photography, dealing with "The mechanism of the latent image," various emulsion problems, questions of sensitivity, and sensitometry. Optical subjects are dealt with in Sections 1 *b* and 1 *c*, opening with an interesting paper on "Photography and the Wave Theory of Light" by Mr T. Smith. There is also an admirable paper on "The Photography of Sound Waves," by Mr N. Fleming, with some excellent photographic records. Spectrophotometry is adequately treated, and Mr H. W. Lee deals with "The Modern Super Speed Lens." Some useful practical papers on printing processes and the reversal of negatives are included in the second part of this section. Photo-engraving and photomechanical processes generally occupy a considerable part of Section 1 *d*, and to those interested the papers will prove useful and informative. The paper on "The Chromium Facing of Printing Surfaces," by Mr E. A. Ollard, is particularly opportune, as it deals practically with a subject on which very little information is available. Colour photography has received due attention, and there are papers on Kinematography which deal with sound films or talking motion pictures. Dr Walter Clark fully explains the International Classification of Photography and gives a draft of the scheme. Altogether this report of the Congress proceedings forms an invaluable work of reference and a very complete presentation of the position of photography to-day.

W. G.